Chapter 1

Resonances Everywhere

Alien View of Earth

If an alien visitor from a distant planet came to visit the Earth, there are certain prominent features of life on Earth that would immediately strike the alien as significant characteristics of terrestial life, but that pass almost completely unnoticed to human eyes due to their overwhelming familiarity and ubiquity. First of these is a fundamental symmetry and periodicity seen in virtually all life forms, like the examples in Figure 1. We see symmetry in the starfish, the octopus, pine trees, palm trees, and in the splendid diversity of flowers. We see periodicity in the segments of a worm, the ribs and vertibrae of a snake, the scales on fish, feathers on birds, the periodic pores and hairs on a mammal's hide, and in the patterns of veins in leaves and stems of plants. What is the origin of this symmetry and periodicity in nature? What is the pattern or template that nature uses to sculpt these symmetrical and periodic patterns in living tissue?

Figure 1. Symmetry and periodicity abound in the forms of plant and animal bodies on earth.

We now know the reason for this pervasive principle of periodicity and symmetry in living things. The mechanism behind it is a *chemical harmonic resonance* known as *reaction diffusion*, a continuous, circular reaction that produces a periodic pattern of chemical concentration, and it is the natural periodicity and symmetry of reaction diffusion that explains the pervasive symmetry and periodicity in nature.

The principle of reaction diffusion is demonstrated most clearly in the Belousov-Zhabotinsky reaction. This involves a continuous chemical reaction in which some reagent A is continuously catalyzed into a different reagent B, while at the same time reagent B is being continuously transformed into reagent A. The net result depends on the relative concentrations (and diffusion rates) of the two reagents, a high concentration of A produces more reagent B, whereas a high concentration of B produces more A. The chemical state is thus unstable, and will cycle endlessly between high concentrations of A and B alternating over time, and this is visible in the Belousov-Zhabotinsky reaction as a cyclical change in color from light to dark and back again, as the reaction proceeds.

Chemical diffusion keeps the chemical state uniform throughout a small sample of the Belousov-Zhabotinsky reagents, thus all of the molecules within that sample tend to change state in lock step. In a larger vessel, chemical diffusion takes time to propagate through the volume of the chemical bath, so the synchrony to more remote parts of the chemical is phase lagged by the time it takes for diffusion to propagate between those points. So reaction-diffusion strikes a dynamic balance between the competing constraints of cyclical reaction at every point in the fluid, and local chemical synchrony enforced by diffusion, which propagates at some fixed rate. The dynamic solution to these constraints appears as a periodic spatiotemporal pattern of chemical waves that emerge spontaneously from the chemical brew, and propagate in parallel fronts through the volume of the liquid in such a way that each point in the fluid cycles endlessly between concentrations of A and B as the chemical wave fronts pass through that point. Figure 2 A shows the patterns obtained by the Belousov-Zhabotinsky reaction evolving over time in a beaker, and Figure 2 B shows the reaction confined in a test tube.

The pattern of chemical waves in the beaker in Figure 2 A is largely a feed-forward phenomenon of waves propagating outward in concentric rings from spontaneous ignition points. In the test tube in Figure 2 B, where the reaction is more tightly constrained in a cylindrical vessel, the reaction-diffusion tends to form a periodic pattern of stripes along the length of the tube, the only direction that diffusion is free to propagate any significant distance. If the height of the column of chemical liquid is contrived to be an integer multiple of the natural wavelength of the Belousov-Zhabotinsky reaction, it is possible to establish standing waves of chemical harmonic resonance, in which each point in the liquid cycles endlessly,

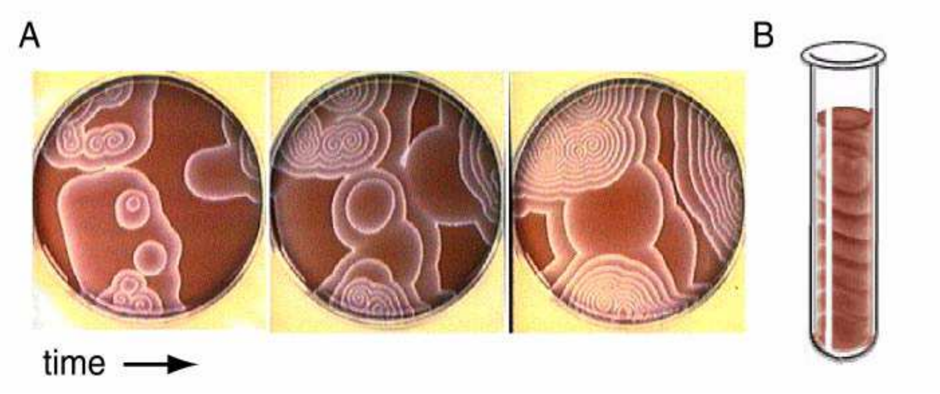

Figure 2. A: The Belousov-Zhabotinsky reaction evolving over time in a beaker, and B: defining a striped pattern in a test tube.

but the pattern as a whole remains stationary, oscillating about a set of fixed nodes disposed at periodic intervals along the tube. It is by this kind of chemical standing waves that nature defines the periodic patterns of tissue in the geometrical designs of plant and animal forms.

Figure 4 A shows an embryo of the fruit fly *Drosophila* at an early stage of development when the periodic pattern of its segmented body is first being laid down, as revealed by appropriate staining techniques that render the pattern visible. These patterns of alternating chemical concentration fix the fate of the underlying tissue, transforming the initially undifferentiated tissue into layers of specific tissue types. Tissue that is surgically transplanted from one part of the embryo to another *before* this critical period, takes on the character of the destination tissue into which it is transplanted, whereas tissue that is transplanted *after* this critical period, retains its original tissue identity at the transplanted location. The template for the periodic striped pattern is a *chemical harmonic resonance* whose standing waves define the stripes in the pattern.

There are actually several sets of such stripes defined by several different reaction-diffusion processes occurring in the embryo, that together define different hierarchical levels of the pattern template for the insect’s body, as shown in Figure 4 B. The highest level of the hierarchy is defined by the so-called *maternal* genes, (whose chemistry is supplied by the pre-natal maternal environment) that distinguish the head end from the tail end of the embryo, or anterior from posterior. The corresponding *morphogens*, (chemicals that determine the morphology of the embryo) which are produced by specialized cells in the head and tail, diffuse passively through the tissue of the embryo, creating a gradient of chemical concentration for each morphogen as shown by the colored shading in the figure.

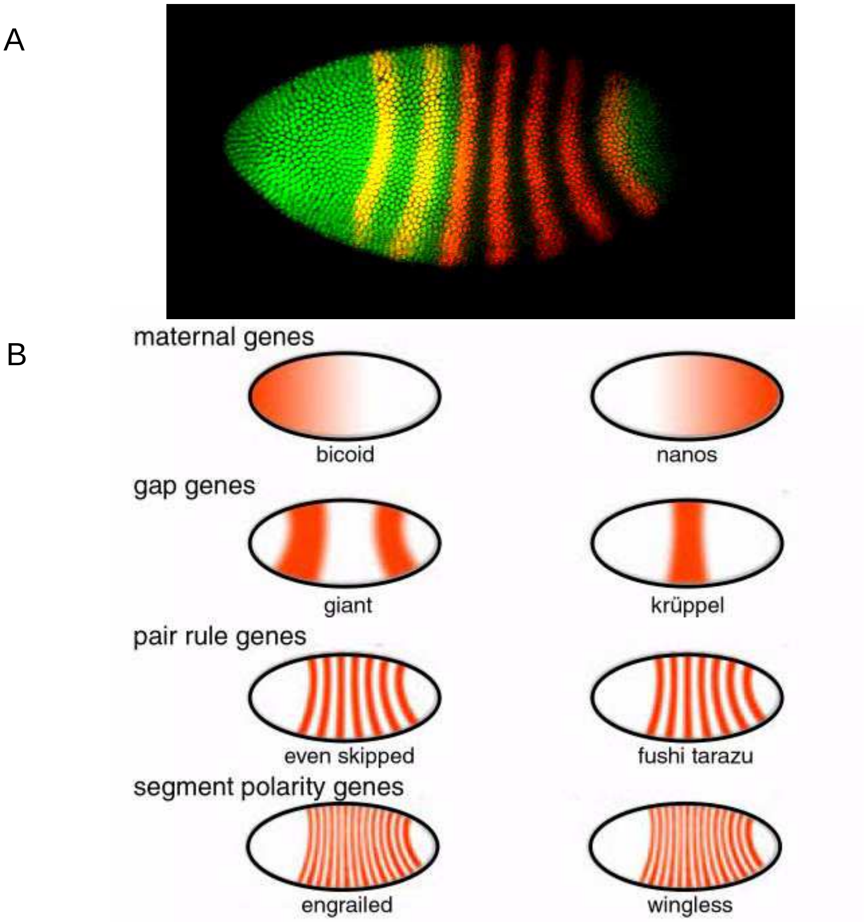

Figure 3. Chemical harmonic resonance in embryological morphogenesis. A: A pattern of chemical concentration in the developing embryo, defined by B: a hierarchical series of component morphogens, each defined by a different standing wave pattern across the embryo.

The next hierarchical level is defined by the *gap* genes, *giant* and *krüppel*. (named for the embryonic deformations produced by mutations that disrupt each morphogen individually) The sharp boundaries between adjacent regions of each morphogen shows evidence of a chemical standing wave like the Belousov-Zhabotinsky reaction, since these boundaries would quickly blur if they were driven by passive diffusion alone, as in the maternal genes. Besides, passive diffusion is an interpolative function that can only generate continuous gradient patterns, it cannot in principle generate the alternating striped patterns of the chemical standing wave. Similar patterns of even finer stripes are defined by the *pair rule*, and the *segment polarity* genes. Considered in combination, these various patterns of chemical concentration uniquely identify or address every point in the embryo as to which segment it is located in. For example the *maternal* and *gap* genes together divide the embryo into five segments each uniquely identified by a specific pattern of concentration of the corresponding morphogens. The *pair* *rule* genes further subdivide those zones into the regular pattern of insect segments, and the *segment polarity* genes divide each segment into anterior and posterior halves. This creates something like a Boolean addressing scheme, in which the relative concentrations of a small number of morphogens determines the specific tissue type required in each segment of the embryo. Figure 4 A is a multiple exposure that superimposes the patterns of three of the morphogens in one picture to show how they work in combination to create a compound pattern of embryonic architecture. This chemical harmonic resonance by way of chemical standing waves is the code used by nature to determine the morphology of the developing embryo, and it is the fundamental periodicity of the phenomenon of standing waves that accounts for the pervasive symmetries and periodicities of plant and animal forms. The fact that nature both *can* and *does* make use of chemical harmonic resonance as a template for defining the spatial patterns of the body in morophogenesis is significant.

At one glance at our bodies, the alien visitor to earth would immediately recognize that we and our fellow fauna and flora are sculpted by harmonic resonance. Besides the linear periodicity clearly evident in the pattern of our segmented vertibrae and ribs, there is a further periodicity in the segments along our limbs, fingers, and toes, with chains of similar bones linked by cartelagenous joints along each individual finger and toe. There is also a radial periodicity evident in the pattern of bones along each limb, that begins with a single bone proximal to the body, (the humerus in the upper arm, and femur in the upper leg), followed by a pair of bones more distally, (radius and ulna in the lower arm, tibia and fibula in the lower leg), and the number of bones continues to increase in stages towards our distal fingers and toes, as suggested schematically in Figure 4 A. This basic pattern is common across a wide variety of species, seen most clearly in early limbed creatures like the Plesiosaurus shown in Figure 4 B. This pattern of radial periodicity has been explained by a chemical harmonic resonance in the embryonic limb bud, that jumps to successively higher harmonics as the limb bud grows in physical size, as suggested schematically in Figure 4 C. (Newman & Frisch 1979)In the human and other mammalian forms, the triple and quadruple bones in the foot and ankle are clumped together into a bony mass due to non-uniform growth rates in different parts of the limb after the initial tissue imprinting during the critical formative period, thus demonstrating that the geometrical regularity of the morphogenic resonance is only present or significant during the critical period in the embryonic limb bud. And the bilateral symmetry of the mammalian body, as well as the four-fold symmetry of a box jellyfish, and the fivefold symmetry of a starfish, are all manifestations of the principles of harmonic resonance in morphogenesis.

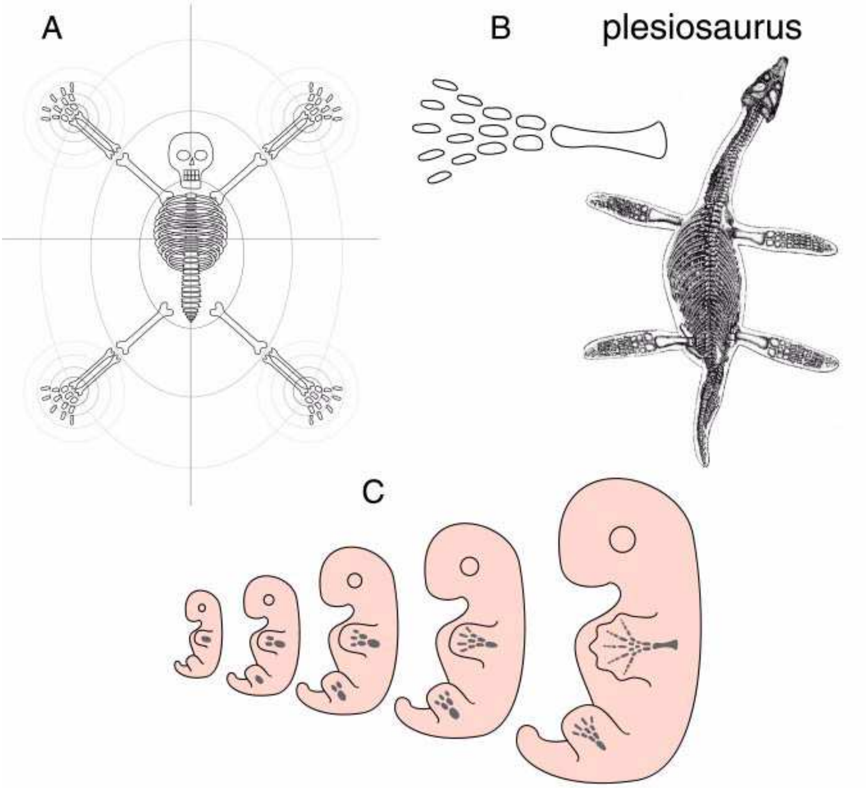

Figure 4. A: A pattern of progressive increase in the number of bones in each limb from proximal to distal, B: seen most clearly in early limbed creatures like the plesiosaurus. C: This pattern is explained by a successive jumping to higher harmonics of chemical harmonic resonance in the embryonic limb bud as it grows to larger size.

There are many more examples of chemical harmonic resonance in morphogenesis, evident just about everywhere that symmetry and periodicity are in evidence. Murray (1981, 1988) shows how the patterns of markings seen on animal skins, such as the spots of a leopard and giraffe, and the stripes of a tiger and zebra, shown in Figure 5 A through D, can all be explained as parametric variations on a single reaction-diffusion mechanism, computer simulations of which are shown in Figure 5 E through H. Murray clinches the argument by showing how the same parameters that produce spots, like those on a leopard, will naturally tend to form rings at the tip of the tail, as is observed for many spotted creatures, due to the long and thin geometry of the tail, by the same principle that the concentric rings in the Belousov-Zhabotinsky reaction in Figure 2 A becomes a striped pattern in the test tube in Figure 2 B. This phenomenon is replicated in the computer simulations of Figure 5 E. Of course the geometry of the tail is only influential during the critical period when the reaction-diffusion pattern is being permanently fixed in the embryonic tissue. Murray also showed how the triangular pattern of stripes on a zebra where the foreleg meets the body, seen in Figure 5 B, can also be explained by the same reaction-diffusion process that creates the more regular pattern of stripes on the leg and the body, as shown in the computer simulation in Figure 5 F.

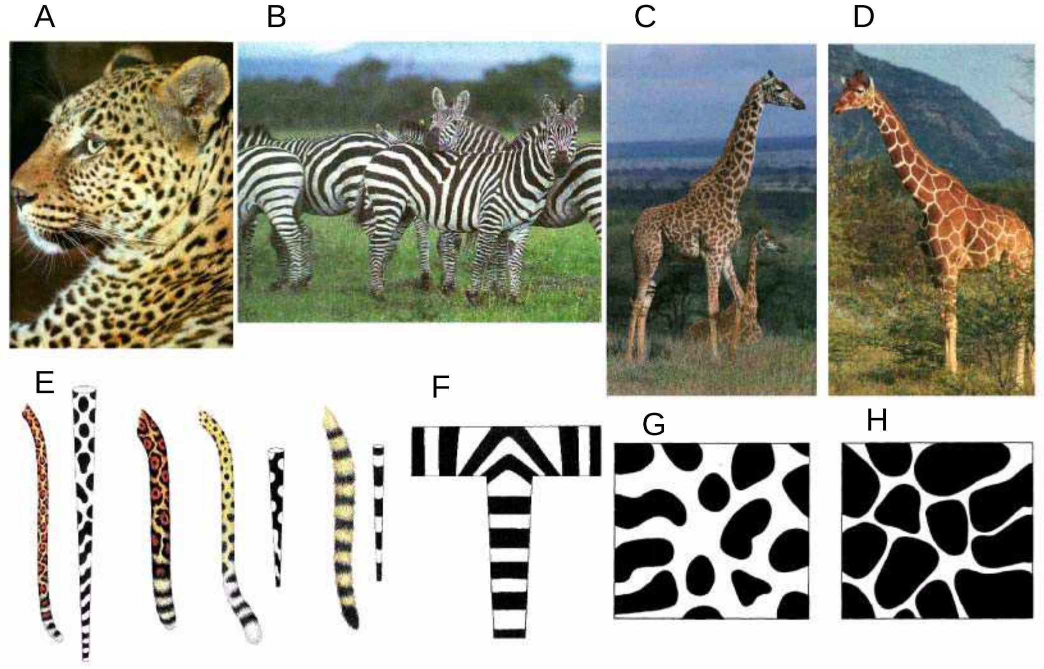

Figure 5. A through D: The great variety of spots and stripes seen on animal hides can be explained by parametric variations of a reaction-diffusion as shown in computer simulations E through H that replicate observed phenomena like the way that E: spots often turn to rings at the tip of the tail, and F: the way the stripes from the forelegs and torso of a zebra merge, and G and H: variations in the spots of different species of giraffes can be produced by parametric variations of the reaction-diffusion.

The patterns on animal hides, which are generally for the purpose of camoflage, demonstrate that reaction-diffusion can generate random, or quasi-random patterns as well as more regular periodic and symmetrical patterns, although the principles of periodicity and symmetry are nevertheless evident as the foundational basis of even these irregular patterns. More regular geometrical patterns are seen on animals whose markings are not intended for camoflage, but as a warning. For example many poisonous snakes and caterpillars, among other creatures, advertise their danger to predators by more prominent geometrical patterns such as stark stripes or rings or diamond patterns.

Why Standing Waves?

Why did nature pick the phenomenon of chemical standing waves as the operational principle behind the pattern formation mechanism of embryological morphogenesis? The answer to this question turns out to be of great significance not only for theories of morphogenesis, but also for other applications in biology where spatial representations are needed, that is, in perception, cognition, and motor control, and one day spatial representation by harmonic resonance will also be a significant human technology once we have figured out how to replicate the central computational principle behind the brain. What makes standing waves so useful as a pattern formation principle is the flexibility or adaptibility of the generated patterns. Unlike a template or blueprint that defines a rigid inflexible pattern, standing waves represent a *multipotential* pattern formation principle that can potentially generate an infinite variety of different patterns that all conform to certain general rules, as seen for example in the infinite variety of leopard spots on individual leopards, all of which exhibit the familiar characteristic pattern for that particular species, and yet each individual exemplar of that infinite variety of leopard patterns is distinct from the infinite variety of giraffe spots on individual giraffes. The secret behind the adaptibility of standing waves is that they are an *emergent feedback* phenomenon that continuously reconfigures itself (during the critical period) to adapt to ambient conditions, like an elastic template or blueprint that can be stretched or squeezed in a number of different directions, while retaining its relative proportions.

For example the segments of the morphogenic patterns in Figure 3 are defined not in terms of a rigid periodic template, but the boundaries between segments automatically shift and distort to create segments of equal *volume* rather than of identical geometry, because it is the volume of each segment that determines its rate of production of morphogen, rather than its geometrical shape. This results in a *topological* rather than *topographical* pattern template, one that automatically adapts to any irregularities or deformations in the embryo to produce geometrically warped segments that conform to the deformation of the embryo. That is why plant and animal bodies, even of the same species, exhibit an astonishing range of sizes, shapes, and varying proportions, while each body is complete with its pattern of limbs, bones, and internal organs all neatly re-proportioned to exactly match the proportions of the given individual. Although the principle of reaction diffusion in embryological morphogenesis is no longer controversial, and is well known within that specialist field, there is little recognition of the profound implications of this highly flexible and adaptive pattern formation principle in nature.

The real power and flexibility of morphogenesis can be seen in the great variety of conjoined twins, whose bodies can fork or branch at any of a number of different locations, as shown in Figure 6, and yet in each case, at the point where the body branches, every bone, muscle, organ, nerve,sinew, and blood vessel, branches neatly at that point, as if that were part of the original plan. This is a remarkable property for a pattern formation template, quite unlike any other spatial patterning principle devised by man, and certainly unlike the plans or blueprints commonly used in architecture and engineering, where the size, shape, and location of every component is rigidly specified to the minutest detail. This remarkable adaptibility of the body plan to variations in body topography is a necessesary prerequesite for evolution, if random mutations that modify the body plan in any way are to have any chance of viability.

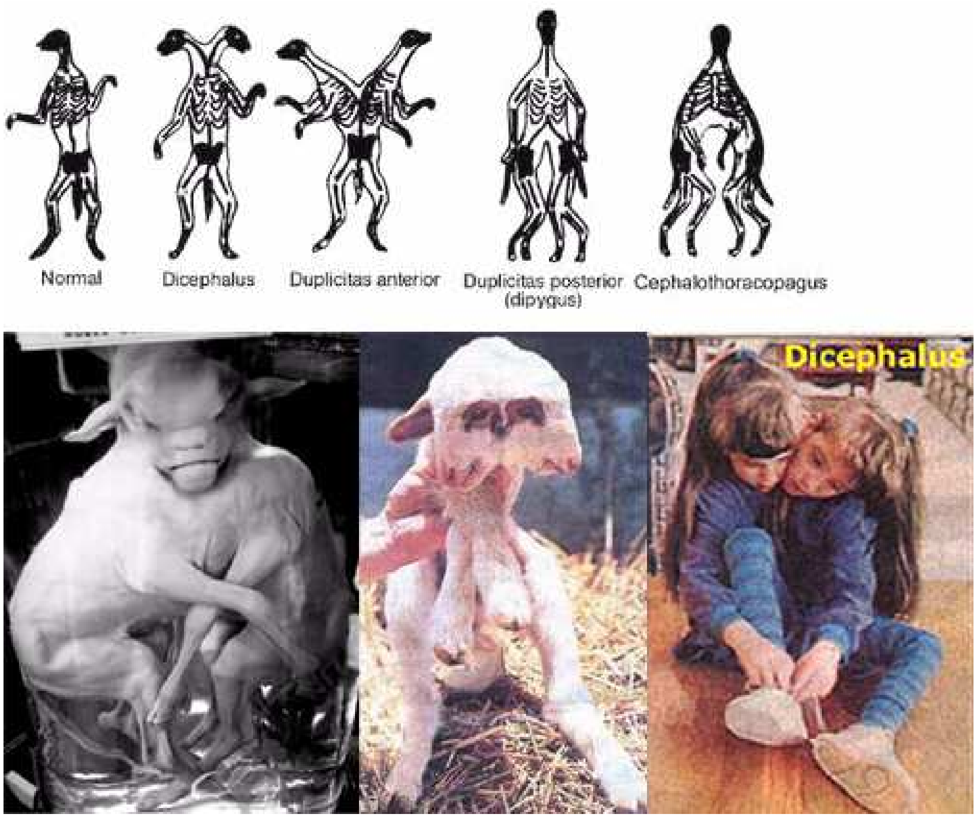

Figure 6. The phenomenon of conjoined twins demonstrates the remarkable flexibility and adaptability of the biological body plan. Wherever the body plan splits or branches, every bone, muscle, organ, nerve,sinew, and blood vessel, branches neatly at that point, as if that were part of the original plan.

Emergence, Holism, and Gestalt Theory

There is something truly magical about the holistic emergent nature of the pattern formation principle observed in embryological morphogenesis. And a similar holistic emergent principle is observed in many aspects of visual perception and of motor control. This was the principal thesis of the *Gestalt* movement of the early nineteenth century, as espoused by the founders of Gestalt theory, Max Wertheimer, Kurt Koffka, and Wolfgang Köhler. The most significant general property of perception identified by Gestalt theory was a holistic, or global-first character in which the global configuration of a stimulus is often perceived before its local component features. Figure 7 shows a picture that is familiar in vision circles, that reveals the principle of emergence in a most compelling form. For those who have never seen this picture before, it appears initially as a random pattern of irregular blotches. A remarkable transformation is observed in this percept as soon as one recognizes the subject of the picture as a Dalmation dog in patchy sunlight in the shade of overhanging trees. The outlines of the dog are defined by a large number of apparently chance alignments of irregular edges. What is remarkable about this percept is that the dog is perceived so vividly despite the fact that much of its perimeter is missing. Furthermore, visual edges that form a part of the perimeter of the dog are locally indistinguishable from other less significant edges. Therefore any local portion of this image does not contain the information necessary to distinguish significant from insignificant edges. This figure therefore reveals a kind of processing in which global features are detected as a whole, rather than as an assembly of local parts.

The principle of emergence, sometimes expressed as the Platonic motto "*The whole is more than the mere sum of its parts*," seems to suggest some kind of magical mystical process whereby perceptual structure appears out of nowhere. However, Wolfgang Köhler (1924) argued that there is no magic in emergence; the principle of emergence is seen in many physical systems, including the way that electric charge distributes itself throughout a conductor, or water seeks its own level in a vessel, where the final position of every particle in the system depends immediately on the positions of every other particle. Perhaps the most familiar Gestalt example of emergence in a physical system is the soap bubble (Koffka, 1935). The spherical shape of a soap bubble is not encoded in the form of a spherical template or abstract mathematical code, but rather the form emerges from the parallel action of innumerable local forces of surface tension acting in unison.

Figure 7. The dog picture is familiar in vision circles, because it demonstrates the principle of emergence in perception. The local regions of the image do not contain sufficient information to distinguish significant form contours from insignificant noisy edges. As soon as the image is recognized as that of a dog in dappled sunshine in the dappled sunshine under overhanging trees, the contours of the dog pop out perceptually, filling in visual edges even in regions where no edges are present in the stimulus.

Principles of Harmonic Resonance

The principle behind the emergent pattern formation mechanism of reaction diffusion can be better understood by examining the properties of other kinds of standing wave resonances, such as vibrational resonances in elastic solids, electrical resonances in electric circuits, and laser and maser systems that operate by standing waves in light and microwaves respectively. It turns out that many of the properties of chemical standing waves are expressed by the same mathematics that describes the behavior of those other resonances. In fact, harmonic resonance and standing wave phenomena represent a higher order organizational principle of physical matter, that transcends any particular implementation of it.

Murray (1981, 1988) demonstrated the link between reaction diffusion and standing wave vibrations of solid objects. Figure 8 (from Murray 1988) shows a thin sheet of steel cut in the approximate shape of an animal hide, that is subjected to vibrations of various frequencies and waveforms to induce vibrational standing waves in the steel, showing how the patterns of standing waves replicate the same kinds of spotted and striped patterns observed in animal markings. Like the Belousov-Zhabotinsky reaction, each point in the vibrating steel sheet cycles endlessly between deflections in alternate directions, like a guitar string when it is plucked. Immediately adjacent points in the vibrating surface are constrained to vibrate in synchrony, whereas the synchrony between more distant points in the sheet are systematically phase-lagged by the time it takes for the vibrational wave to propagate between those points. Normally the vibration pattern of a steel plate is virtually invisible, since the actual deflections are so minute in magnitude, and fleeting in duration. The standing wave patterns in Figure 6 A were made visible by constructive and destructive interference between coherent laser light illuminating the plate, and the light reflected back from the plate, showing deflection in one direction as dark, and in the other direction as light. The dark/light pattern reverses many times each second along with the vibration of the plate, so to capture the pattern photographically, the illuminating light must be strobed in synch with the vibration of the plate, to capture only one phase of the pattern and not the other.

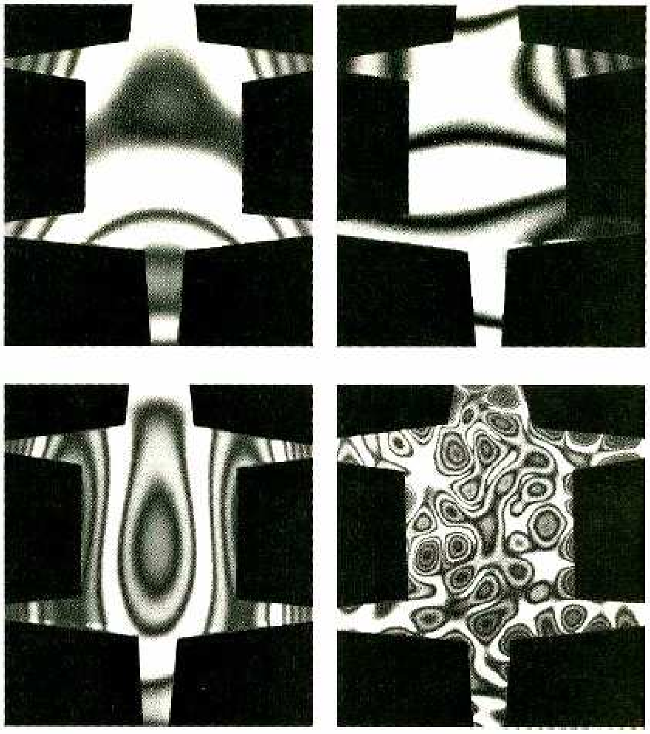

Figure 8. Murray (1988) showed the connection between chemical and vibrational standing waves by replicating the patterns of leopard spots and zebra stripes in the standing wave resonances in a vibrating steel sheet cut in the approximate shape of an animal skin.

Another way to record the patterns of standing waves on a steel plate was devised by Ernst Chladni, back in 1787, illustrated in Figure 9. Chladni sprinkled dry sand or powder on steel plates set into vibration using a violin bow. The sand dances around randomly on the vibrating surface, but eventually settles along the nodes of vibration, the lines of zero vibration that separate patches of the plate that are vibrating in counterphase to each other, thus rendering those node lines visible. Unlike the forced, or driven vibrations of Murray's steel plate, Chladni's bowing with a violin bow energizes the vibration without imposing any particular frequency or waveform on it, so that like a violin string, the plate will automatically resonate at its natural modes of harmonic vibration. This principle of energizing vibrations without imposing a vibration pattern on them, is seen in a wide variety of musical instruments. Different patterns are obtained by touching the plate with a finger at some point, as shown in the figure, which creates a node of vibration at the damped location.

Mary Waller (1961) refined Chladni's technique for producing standing waves in steel plates. Instead of bowing the plate with a violin bow, Waller pressed a piece of dry ice against the plate. As a child she had noticed how a piece of dry ice from her neighborhood ice cream vendor would make the bell of her bicycle ring with a continuous tone or squeal when the dry ice was pressed against it. The carbon dioxide gas that sublimates from the dry ice in direct contact with the steel, opens a tiny gap between the ice and the steel for the gas to escape, which then slams closed again for each tiny bubble of escaping gas, creating a continuous ringing or vibration in the steel. Different standing wave patterns can be generated on a Chladni plate in this manner by applying the dry ice to different points on the steel plate, by pressing harder to generate higher frequency tones, and by touching parts of the plate with a finger to damp the oscillations at various points. Figure 9 shows three plates, one square, one round, and a third one cut in the shape of a guitar soundboard.

Figure 9. Chladni figures produced by bowing a steel plate while damping the plate with a touch of a finger. The patterns of standing waves are revealed by a sprinkling of salt that settles along the nodes where the vibration is minimal. Three plates are shown, one square, one circular, and a third shaped like the body of a guitar.

Figure 10 (adapted from Waller 1961) shows a variety of standing wave patterns obtained for a square plate that is clamped at its midpoint. The patterns toward the upper-left correspond to lower frequencies of vibration of the plate, whereas the patterns toward the lower right in the figure correspond to ever higher frequencies of vibration of the standing waves, which requires higher energies of vibration. Every distinct pattern of vibration of the plate shown in Figure 7 corresponds to a distinct and unique frequency of vibration, that can range from a low baritone note, to a high pitched squeal, eminating from the plate as each standing wave pattern appears.

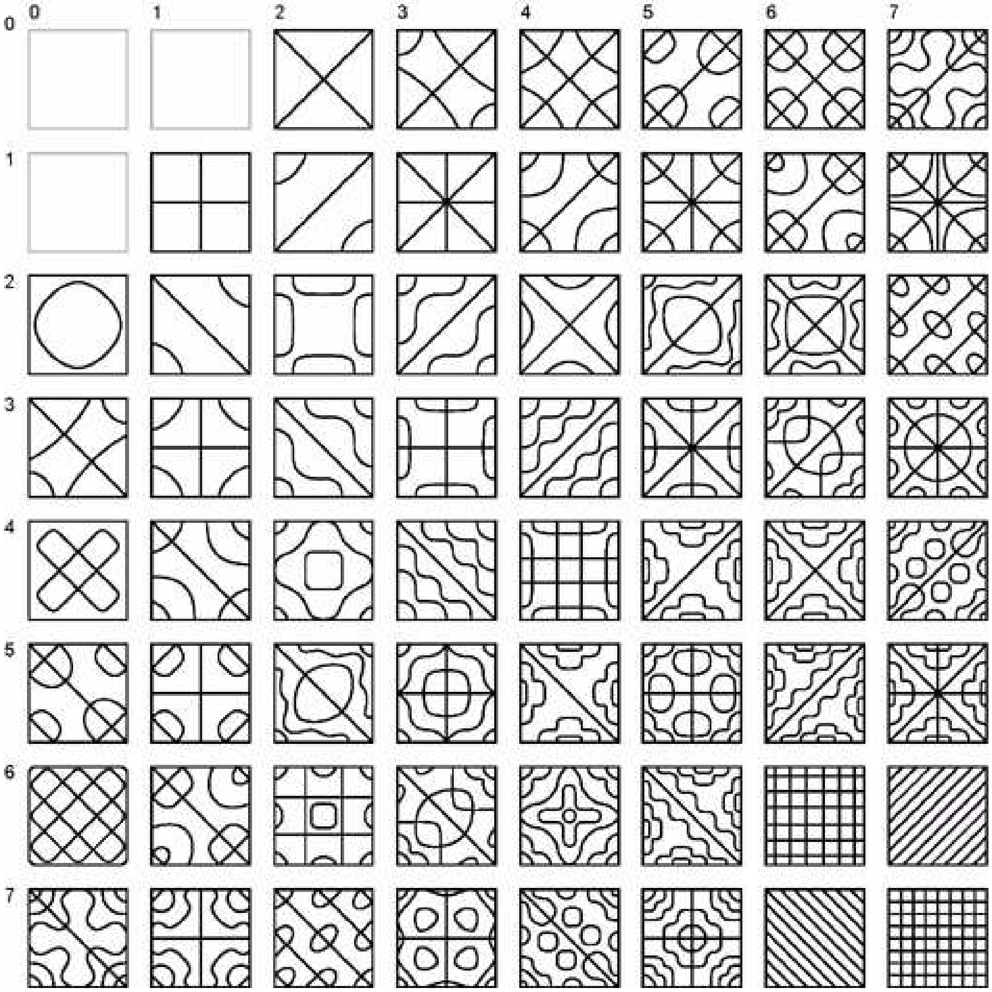

Figure 10. Chladni figures for a square steel plate (adapted from Waller 1961) demonstrate the fantastic variety of standing wave patterns that can arise from a simple resonating system.

The range of patterns observed in Chladni figures depends on the shape of the vibrating plate. Figure 11 shows Chladni figures obtained for a circular plate, with radial and concentric node lines that subdivide the circle into periodic segments.

Chladni figures are the pre-eminent example of emergence. From totally homogeneous medium emerges not one, but a whole family of periodic and symmetrical patterns that are related by integer harmonic relations into a

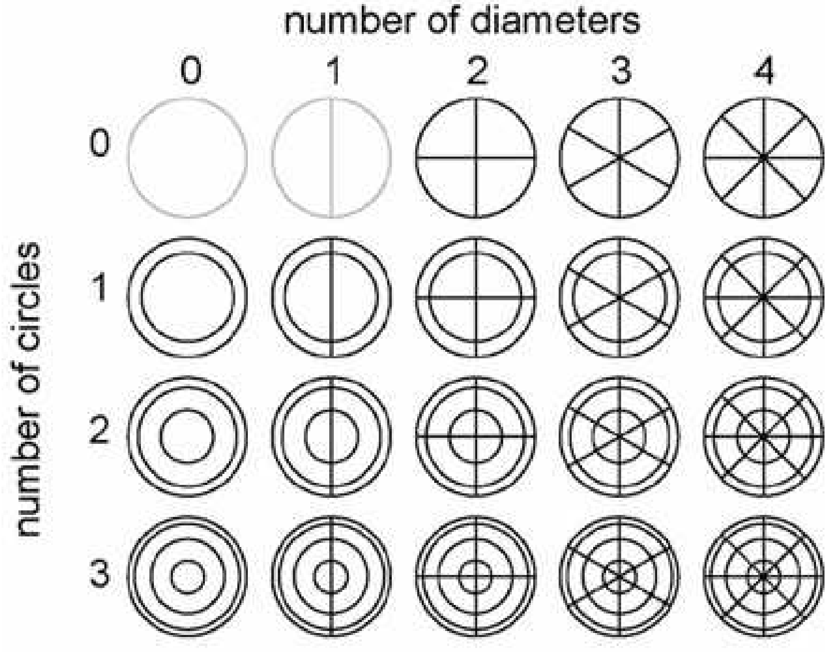

Figure 11. Chladni figures for a circular plate, sorted by number of [diameters, circles] in each pattern. These patterns can appear at any orientation on the plate.

hierarchical family tree, and the geometry of those patterns is not determined so much by the local properties of the resonating medium, as by the global configuration of the resonating system as a whole.

Entrainment and Coupling of Resonances

One of the most remarkable properties of the phenomenon of harmonic resonance is the tendency of resonating systems to couple or synchronize with each other, even when the coupling forces between individual resonances are weaker by orders of magnitude, than the internal forces within each resonator. Christiaan Huygens, who perfected the pendulum clock, discovered the phenomenon in 1665 when he was running two pendulum clocks in parallel for comparison of their timekeeping. Huygens discovered that if one clock ran only slightly faster or slower than the other when hung on opposite walls of a room, those same clocks would run at exactly the same speed when hung two feet apart on the same wall. Not only would they keep the same time, but he found that their pendulums would swing in synchrony, in perfect counterphase, like mirror images of each other alternately approaching and withdrawing. Apparently the tiny vibration caused by the oscillations in each clock was transmitted through the wall sufficient to establish the synchrony between the oscillations through this miniscule synchronizing force. This is the phenomenon of *entrainment*, a central principle of harmonic resonance seen in countless forms throughout the universe.

We see resonnce in the way an electron orbits its atom, and how it reacts with different frequencies of light, absorbing and emitting only at a wavelength that harmonizes with its own resonant frequeny, imprinting a complex harmonic signature in the emmission and transmission spectra of clouds of gas, stars, and galaxies based on their elemental composition. It is a resonance phenomenon that different wavelengths are absorbed very differently by material substances. Low frequency infra-red radiation are absorbed by whole molecules, whereas higher frequency x-rays are absorbed by the atoms of those molecules, and gamma rays are absorbed by the nuclei of those atoms. Resonance is the principle by which radio and television signals which are brodcast at many different frequencies, can be tuned for individually by a radio receiver whose resonant frequency can be tuned to receive only a single channel at a time. And it is a resonance principle that sorts Saturn's rings into patterns of bands, and that distributed our planetary orbits to follow Kepler's law, whereby the *squares* of the periods of the planets are proportional to the *cubes* of their semimajor axes. In all of these examples of entrainment of oscillations, we see a miniscule force, applied repeatedly at just the right frequency to an oscillator system, to produce a quite considerable response.

Resonance also comes in more complex compound form with coupled oscillators. For example if two or more pendulums are hung from a horizontal cross-string, then their motions will be weakly coupled by the cross-string. If one pendulum is set to swinging, then the tiny coupling force will slowly set the next pendulum to swinging faster and faster, even as the first one goes ever slower and slower, until eventually the second pendulum will be swinging at full amplitude and the first one has been brought to a complete standstill, at which point the situation reverses, and the first pendulum gradually robs energy from the second until the original condition is restored. This pattern can continue indefinitely, each pendulum taking turns in a cyclic sequence. If there are four pendulums hanging from the cross-string, then there are several different characteristic modes at which they can oscillate. They can all swing in unison, like a single virtual pendulum. Or they can swing in pairs, with the first two in phase with each other, and in counterphase to the last two. Or they can swing in alternate pairs, the first and third swinging against the second and forth. These are the natural modes of four coupled oscillators, each of these patterns is stable, that is, it will continue indefinitely if once set in motion, at least in the absence of frictional losses, or if the oscillation is amplified. So again, we see emergent complexity arising from a very simple system of identical oscillators, natural patterns of complex behavior in the absence of any explicit pattern generation mechanism. In mechanical systems like a car engine, this kind of pattern of sequenced motion is often enforced by cams and push rods, like those that control the valves to open and close at the proper time for each cylinder. In computer algorithms patterned sequences of this sort are usually achieved by loops and counters and logical IF statements. In either case, the complexity of the sequencing mechanism or algorithm is of the same order as the complexity of the pattern of motions that it generates. Emergence is a natural property of harmonic resonance, whereby a very simple homogeneous mechanism composed of identical units can generate patterns whose complexity far exceeds the complexity of the mechanism that generates them. And it is that property of emergence which explains why nature makes use of harmonic resonance as its central pattern formation mechanism in space and time.

I propose that it is harmonic resonance that couples the electrical activation across the whole brain into a single synchronized harmonic oscillation, in which individual specialized brain areas each contribute their component of the oscillation. But the resonance in every brain area is instantly and immediately influenced by the resonance in every other brain area simultanously, like the different parts of a musical instrument sharing the vibraton of a note being played. The vibration is like a ghostly pattern superimposed on the tissue of the cortex, although moving somewhat independently of that tissue, so as to be near invisible to a tissue-anchored electrode, manifesting itself only as an otherwise mysterious synchrony between distant cortical regions. It is this synchrony, the holistic emergent intercoupling of innumerable parts into an integrated whole, that is the explanation for the unity of conscious experience, for the simultaneous and parallel experience of our visual field, our body field, and the space around us, all at once and in fixed relation to each other in a single space. This is how the unity of experience can be resolved with the apparently fragmentary architecture of the cortex, with its specialized areas tuned to individual modes and modalities. Specialized cortical areas do not process their data independently in an input-output fashion, and then transmit the result back to Central Control, as in a computer algorithm, but rather, each cortical area is like a resonator tuned to enhance or amplify the particular feature for which it is tuned, whether a spatial pattern in visionm, or a temporal pattern in music or audition, or a spatiotemporal pattern in somatosensory and motor representations. When that area resonates with a recognized feature, an echo of that resonance is felt in every other cortical area, each of which resonate with their own contribution to the global resonance shared by them all, in the same way that the vibrational resonance at every point in a guitar is influenced by a finger pressed against a single string as it vibrates. This was the principal message of Gestalt theory: that the unity and coherence of experience implicates a unitary and coherent principle of perceptual computation in the brain, and harmonic resonance is that principle of organization in the brain.

The spatiotemporal coupling between cortical areas in the brain is analogous to the coupling between various parts of a Chladni plate. For example pattern [0, 7] (expressed as [row, column]) in Figure 10 appears as two copies of the same pattern reflected across a diagonal symmetry axis, and that is because the standing waves of those two portions of the steel plate are exactly in this kind of symmetrical counterphase, exactly mirroring and thus balancing each move of the other, like two mirror-image copies of the same dynamic patter.

Computational Function Served by Neural Standing Waves

How can harmonic resonant standing waves serve a computational function in the brain? What kind of things can resonance compute? The answer to this question can be found in one of the simplest single-celled organisms from the lowest branches of the evolutionary tree, the *paramecium*. The single-celled Paramecium has little hairlike cilia around its tiny cigar-shaped body, and it waves those cilia in synchronized waves from the head toward the tail, as shown in Figure 12 A, in order to propel itself forward through the water. This global synchronized pattern reveals the presence of global waves of energy propagating down the body of the Paramecium, as suggested in Figure 12 B, and those waves perform a vital integrative control function that synchronizes the motions of the many individual cilia to a globally coherent motor plan. The paramecium is capable of swimming faster or slower, steering in different directions, and when it gets stuck in a blind alley, it can reverse its swimming movements and back out, and turn aside to set off in a new direction. The integrated lawful behavior of this dynamic travelling wave structure is the living soul that animates lifeless tissue with patterns of willful motion. It is a dynamic spatial structure that appears as if out of nowhere, is composed of nothing but patterns of energy, and it disappears without a trace when the tiny creature dies, leaving a lifeless corpse that is not even the tiniest microgram lighter for the loss of its living soul. And yet while the creature is alive, it fairly pulses and throbs with vibrant energy in regular structured patterns of coherent motion under a kind of intelligent or directed control.

The same kind of synchronized waving of cilia is seen also in multi-cellular creatures like jellyfish. Ctenophora, or comb jelly, is a simple creature with a decentralized nervous system, and the comb jelly uses the same synchronized

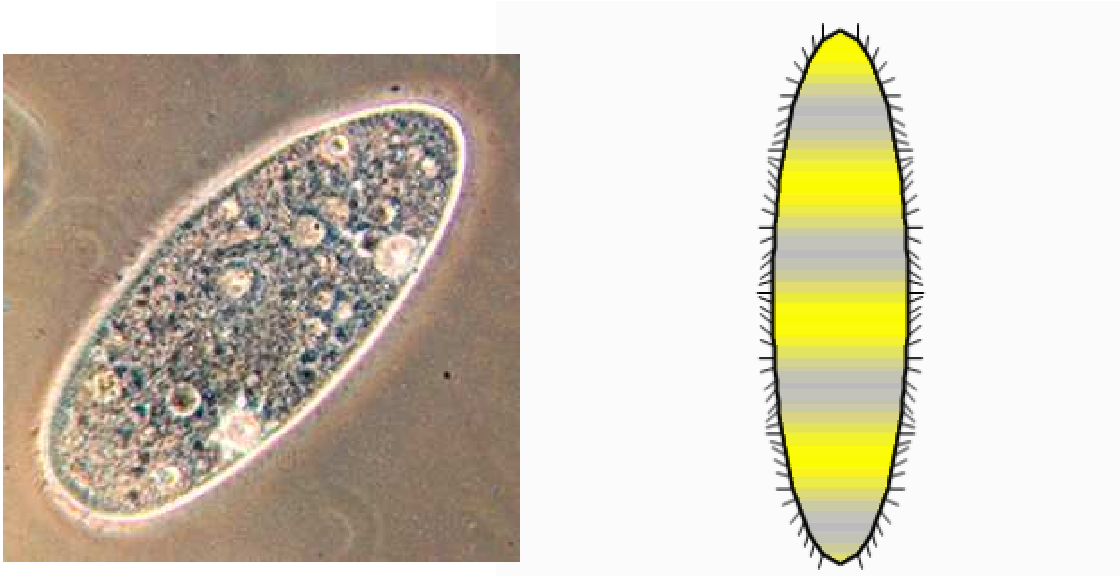

Figure 12. A: The paramecium is a single-celled organism covered with tiny microscopic hairs called cilia, that it uses for locomotion. B: The cilia beat in synchronized waves from head to tail, revealing waves of synchronized activation sweeping across the tiny creature's body.

waving of its cilia for propulsion as does the paramecium, only implemented at a very much larger multi-cellular scale. The waves of contraction sweep along the body of the creature as it swims, and in some species these waves of contraction are accompanied by waves of glowing bioluminescence that sweep over the creature's body, a visible trace of the electrochemical waves that control the propulsive undulations of the cilia, revealing the tiny creature's "motor thoughts", that stream endlessly like the moving letters on an old fashioned theater marquee. This same principle of synchronized waving of cilia exists also in the lining of our own trachea, where the cilia serve to waft mucus and trapped dirt upward from our lungs. The waves of contraction and extension of the cilia stream through the tissue that sustains it without respect to the boundaries of the cell wall.

Similar global patterns of alternating contraction and extension are seen in larger and more advanced animals, such as swimming snakes or eels, like the one shown in Figure 13 A. , The motor pattern of a swimming eel clearly appears as a travelling wave propagating continuously from head towards tail, periodic in space and time. At any one moment, the pattern is defined by a sinusoidal alternation between equally spaced regions of contraction and regions of extension, as shown in Figure 13 B, where the cylinder represents a map of the eel-s quasi-cylindrical body, and the different shades represent regions of contraction and extension mapped to the body. Regions of contraction on one side of the body are balanced by regions of extension on the opposite side of the body, creating

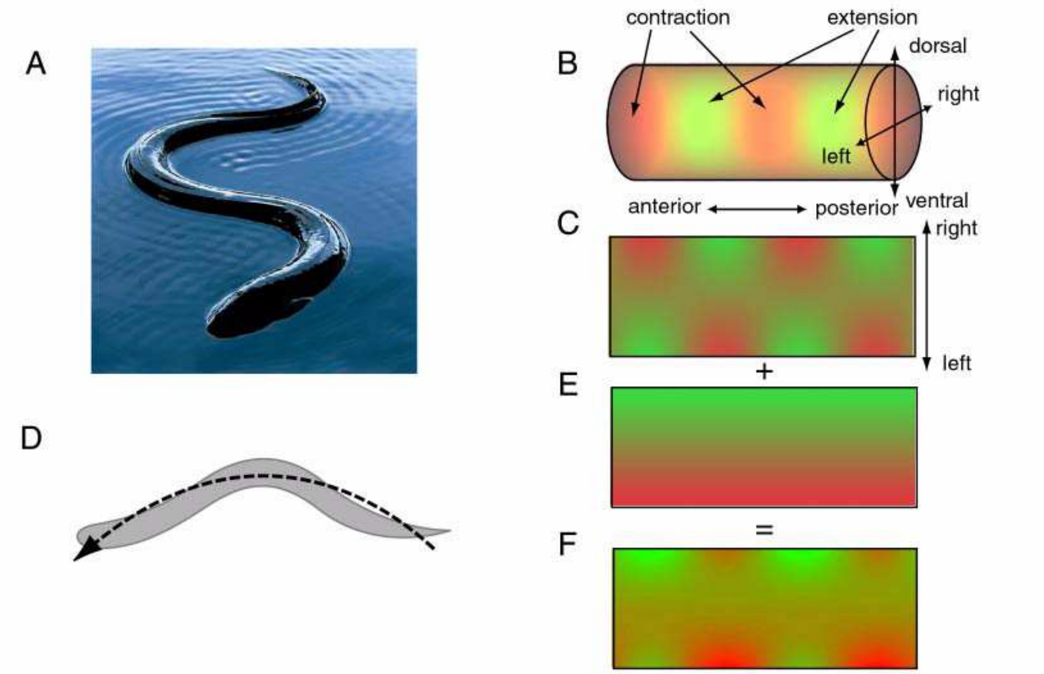

Figure 13. A: The motor pattern of a swimming eel takes the form of waves of contraction and extension on opposite sides of the body, and these waves stream continuously from anterior towards posterior.

polarity-reversed patterns of contraction and extension across the body midline, as shown in the top view of the motor pattern in Figure 13 C. The eel's motor pattern exhibits a perfect symmetry, or balance across the body midline, with a smooth and continuous change of body tension as a function of spatial location. And every instant this pattern translates continuously from anterior toward posterior, like the travelling waves in a rope when one end is shaken up and down vigorously. This dynamic sinusoidal symmetrical pattern is the required "output" of the motor computations of a swimming eel, in order to account for the observed sinusoidal flexion of the body.

The eel can also steer its body in smooth curves left or right, as suggested in Figure 13 D, and also up and down, while still maintaining its sinusoidal propulsive undulations. In order to achieve this, the eel's motor pattern must combine the propulsive undulations of Figure 13 C with a superimposed steering pattern as shown in Figure 13 E, where the motor pattern to steer left is expressed as a contraction along the entire left side of the body, and a symmetrically opposed extension on the right side. This uniform lateral gradient is combined with the propulsive undulations to produce a compound or combined motor pattern shown in Figure 13 F. Note how the periodic regions of contraction are amplified on the left side and diminished on the right, and vice-versa for the regions of extension. More generally, each component of the motor pattern, propulsion faster or slower, steering left or right, and steering up or down, can be controlled independently, and yet they are all combined in such a way that each muscle knows exactly how much to contract or extend depending on its location within the global motor pattern at each instant in time. This poses a formidable computational challenge for any paradigm of motor control. It just so happens that standing waves in a cylindrical resonator or acoustical cavity, have exactly these kinds of periodic symmetrical patterns, and standing waves naturally tend to combine in exactly this manner to produce compound patterns. The ability of snakes and lizards to freeze in the middle of any serpentine motion, and to hold that posture ridigly, clearly demonstrates that the brain can sustain static motor patterns at will, a standing wave rather than a cyclic travelling wave phenomenon. Harmonic resonance has that property also, that is, a natural tendency to form patterns of standing waves, expressed in periodic and symmetrical primitives or basis set.

The necessity for a pattern generating function in motor control is nowhere more evident than in some of the surprisingly complex behaviors observed in creatures with the simplest unstructured nervous systems. For example consider the hydra, a tiny microscopic organism with a simple network for a nervous system, somewhat like a fishnet stocking that extends uniformly across the creature’s body and around each tentacle. The primitive synapses that form between these neurons transmit activation in both directions, so that stimulating the tiny organism at one point creates waves of activation spreading outward from that point, like the rings in a pond when a stone is thrown in, as depicted in Figure 14 A. And yet, with this simple unstructured nervous system the hydra performs some pretty remarkable motor patterns. When it snags food with one of its tentacles, it knows which side of the tentacle to contract more, in order to steer the food toward its mouth, and it knows to open its mouth in anticipation, as suggested in Figure 14 B, and to close it again after engulfing the food. The hydra also performs peristaltic contractions that propel the food down toward the foot, and reverse contractions to expel the undigested remains back out of its mouth. The hydra is also capable of a kind of somersaulting locomotion depicted in Figure 14 C, where it contracts all the muscles on one side of its body and tentacles, leans over and grabs the ground, then it reverses its pattern of contraction and extension to flip upside-down overhead in a somersault, and it repeats this extension / contraction grab / release periodic cycle of motor pattern to travel along a solid surface. How, but for some kind of standing wave model, could a simple unstructured nervous

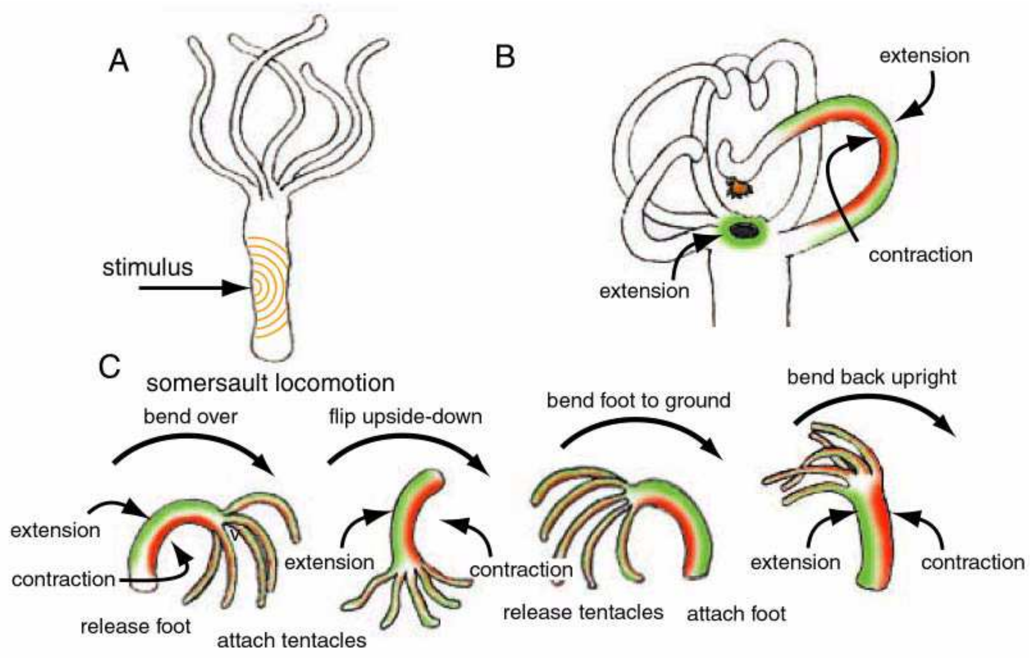

Figure 14. A: The uniform network of a hydra's nervous system propagates electrical activity out in all directions from a stimulation point, and yet this unstructured nervous system is capable of some rather sophisticated spatial behavior such as B: steering captured food from a tentacle into its mouth, which it opens in anticipation, and C: the hydra is capable of a somersaulting kind of locomotion that requires alternating regions of extension and contraction of its body.

system composed of a uniform network of undifferentiated cells, possibly generate such a rich repetoire of motor behavior? The synchronized waving of tentacles, so commonly seen in the simplest animate creatures, is a give-away clue to the wave-like nature of that computational principle. More clues to the harmonic resonance nature of motor patterns is seen in the periodic / symmetrical cycle of the legs of a walking centipede, and the periodic mirror-symmetric stepping motions of a six-legged insect. And evidence for harmonic resonance is seen even in the motor patterns of mammals. Strogatz & Stewart (1993) show that the various gaits of four-legged creatures, for example the trot, canter, and gallop of a horse, correspond to the various modes of oscillation of four coupled oscillators, while the gaits of bipedal creatures correspond to the modes of two coupled oscillators.

The alien visitor to earth, on seeing the rhythmic galloping of a horse, the periodic undulations of an eel, or the tiny waves of cilia on a paramecium, would immediately recognize that we and our fellow fauna employ a harmonic resonance principle of motor control. But then of course, that is what the alien would probably have expected, given that it too is most likely controlled by a harmonic resonance motor principle.

Harmonic Resonance In Sensation and Perception

The powerful integrative capacity demonstrated by harmonic resonance in motor control is just as useful in sensation and perception too, where a spatial world is modeled using some kind of spatial representation. How are the individual sensory stimuli for example from the countless photosensitive cells of the retina, meaningfully integrated into a perceptual picture of the world? Hashemiyoon et al. (1993) have measured rapid electrical oscillations in the retina whose waveform is modulated by the visual imput. I propose that these oscillations are a manifestation of standing and/or travelling waves across the retina which are an essential component of the mechanism that encodes the visual world in a meaningful code. I propose that the retina functions somewhat like a Chladni plate, with a natural tendency to sustain standing waves of electrochemical oscillation across the retinal surface to represent static patterns, and travelling waves to represent dynamically moving or cycling patterns. The visual input on the retina serves to modulate those standing waves so as to produce standing wave patterns that conform to the pattern of light projected on the retina, as a spatial effigy of that stimulus pattern constructed of standing or travelling waves. Unlike the individual sensory signals detected by the innumerable rods and cones, a standing wave is not a mere assembly of parts, but a highly integrated holistic Gestalt structure that cannot survive the elimination of any of its component parts without radically altering its form, because the structure of a standing wave is a continuing and sustained dynamic balance between mirror-image portions of the wave colliding and rebounding cyclically against each other.

The standing waves on a Chladni plate are controlled or modulated by damping the vibrations at different points with a touch of a finger. I propose that the visual input serves this damping function, damping the vibrations along the boundaries or contours detected in the image, dividing the image into contrasting regions of figure and ground, represented by regions of opposite polarity of the standing wave. For example an image of a white square on a black background would be represented by a standing wave of the same shape, with a positive square on a negative background, alternating cyclically with a negative square on a positive ground as a spatial standing wave. The nodal contour serves as the eigenfunction of the image pattern, the line along which the image is neither white nor black, just as in the standing wave it is neither positive nor negative, but serves as a pivot about which the positive and negative regions alternate. As in the patterns of motor control seen in the cilia of the paramecium, the standing wave becomes a distinct spatial entity energized and sustained by the retina, but existing as if in a separate plane of existence than the physical mechanism that sustains it, with its own integral pattern of internal structure and dynamic logic. It is the dynamic logic of standing wave resonances that account for the Gestalt laws of perceptual grouping.

Sensation involves a mere recording or replication the pattern of the stimulus, whereas perception involves a recognition or understanding of the structure of that stimulus, an intelligent analytical function. The standing waves in the retina serve as the first stage of this analytical function, and it is for that reason that vision is particularly sensitive and responsive to periodic and symmetrical patterns, which is why we paint items in starkly contrasting periodic patterns when we want them to be most visible, like the stark barber-pole stripes on a railroad crossing barrier, or the black and white pattern of the “checker flag” used in auto racing, or the vertical stripes of an umpire’s uniform, and so forth.

The resonance seen in the Chladni figures also offers a clue as to how the standing wave patterns are recorded or registered in the brain in a meaningful code. Every unique Chladni plate figure is associated with a characteristic frequency and waveform, composed a fundamental frequency and a pattern of higher harmonics, and every time that pattern appears on the plate, its characteristic voice is also to be heard. Furthermore, if a pattern’s characteristic tone is played in the presence of the Chladni plate, it will have a tendency to vibrate the plate in sympathy with that tone, thus recreating the original standing wave pattern back on the plate. I propose that the relation between a Chladni figure and the harmonic waveform or musical chord heard whenever that figure is present on the plate, is the same relation seen in visual recognition between a pattern in our experience, and our perceptual or cognitive recognition of that form. If spatial experience is constructed of spatial standing waves in the brain, the presence of any recognizable shape in our experience will be accompanied by the characteristic tone of that pattern due to the vibration of the sensory standing wave, and it is by that tone that the shape is registered as a familiar shape in the brain.

Visual recognition is often described in neural network models as the “lighting up” of a node, or neuron in the brain, in response to the presence of its characteristic pattern in the visual stimulus. Although this “grandmother cell” or “feature detector” concept of visual recognition is now largely discredited, at least in its most simplistic form, no clear alternative has been proposed for the essential principle behind visual recognition, and the neurophysiological literature is still replete with references to feature detector cells and their supposed receptive field profiles. The acoustical tones associated with the Chladni figures offers an alternative model of visual recognition. The “feature detector” nodes in the Chladni plate model can be conceived as a bank of tuned resonators set up near the plate. Like a “grandmother cell”, each resonator will spontaneously resonate, or “light up,” whenever its particular pattern is present on the plate. There is a *dimensional reduction* that occurs between the two-dimensional spatial pattern that appears on the plate, and the one-dimensional activation of the resonator that reacts to that pattern, like the reduction between a two-dimensional stimulus pattern and its corresponding “grandmother cell” feature response. But the resonator does more than merely register the presence of its pattern on the plate. What makes the resonator a *meaningful* representation of that pattern is its ability to regenerate its characteristic pattern on the plate, which it does spontaneously and automatically simply by vibrating at its characteristic frequency. The mere activation of a resonator, an essentially one-dimensional event, expands into a two-dimensional standing wave, due to the automatic reification of the spatial standing wave from the simple resonant tone. It is this *constructive*, or *generative* function of visual perception that is prominently absent from so many theories of visual recognition.

Aesthetic Resonance

There is another factor that would be immediately apparent to the alien visitor to Earth, to which we humans seem to have a peculiar blindness, and that is a basic symmetry and periodicity across a vast range of human aesthetic creations, from music, to dance, to ornamental patterns, Gothic cathedrals, Islamic mosques, churches and palaces everywhere. Some of this symmetry can be accounted for by engineering considerations that favor the regularities of a an arch, dome, or vault, as well as of straight planar walls with right-angled corners, and flat floors laid with arrays of identical tiles. But great places of worship take symmetry and periodicity far beyond mere functional necessity and turn it into a joyous celebration of symmetry and periodicity for its own sake, in both simple, and compound hierarchical form. So many of the articles that we make or use are decorated with periodic and symmetrical patterns in both simple and compound hierarchical forms. We decorate our clothing, our wallpaper, pots, and vases, with all kinds of periodic and symmetrical patterns, especially objects of particular religious or ceremonial significance, such as crowns and scepters, crosses and chalices, cutlasses and candelabras. And even those patterns that we enjoy for their simplicity, such as the circle, the ellipse, the heart shape, the equilateral triangle, the regular hexagon, it is their regularity that we enjoy, because the lawful elegance of a regular curve reflects a more general concept of symmetry as a continuation of the same pattern by the same rules. And the periodicity of a polygon is seen in the rotational periodicity of its vertices, the most primal polygons being those with the smallest number of vertices, such as squares and triangles, or no vertices at all, like the circle. And there is periodicity evident at the core of our number system, the periodicity of the concept of number itself, an item that cycles endlessly in identical units that are counted in groups of tens, and then tens of tens, and tens of tens of tens, in a hierarchical pattern that is directly borrowed from the principles of harmonic resonance itself. The concept of the rational fraction, like the subdivision of the unitary interval into the equal intervals of halves, quarters, eights, and sixteenths, or into tenths, hundreths, and thousandths, corresponds directly to the higher harmonics of a fundamental resonance, as seen for example in the standing waves in the body of a bugle as different notes are blown. Only an alien to human culture is likely to notice this pervasive appearance of symmetry and periodicity in human affairs, exactly because it is so pervasive and ubiquitous that we don't even notice it as anything special at all.

Figure 15 shows a variety of ornamental patterns from a variety of different cultures and historical periods, showing the common principles of symmetry and periodicity in simple and compound hierarchical forms. The alien visitor, on seeing these kinds of patterns throughout human art and architecture, would immediately conclude that humans have *harmonic resonance* brains, that is, our brains operate by generating periodic and symmetric patterns of standing waves across the neural tissue, and those standing waves are the principle by which spatial structure is represented and experienced in the brain.

Another manifestation of harmonic resonance in human affairs is seen in human music, with its harmonies and melodies and rhythms, patterns of symmetry and periodicity across tone and time, in both simple and compound hierarchical forms. Music is nothing other than an elaborate system of resonances of resonances of resonances in fantastically complex hierarchical arrangements. Even the basic tones of which music is composed, a sustained note on a bugle, or a violin, or didgeridoo, sounds beautiful to our ears, and that beauty is the beauty of raw resonance itself. Pairs or triplets of tones that are *consonant* , that is, they sound

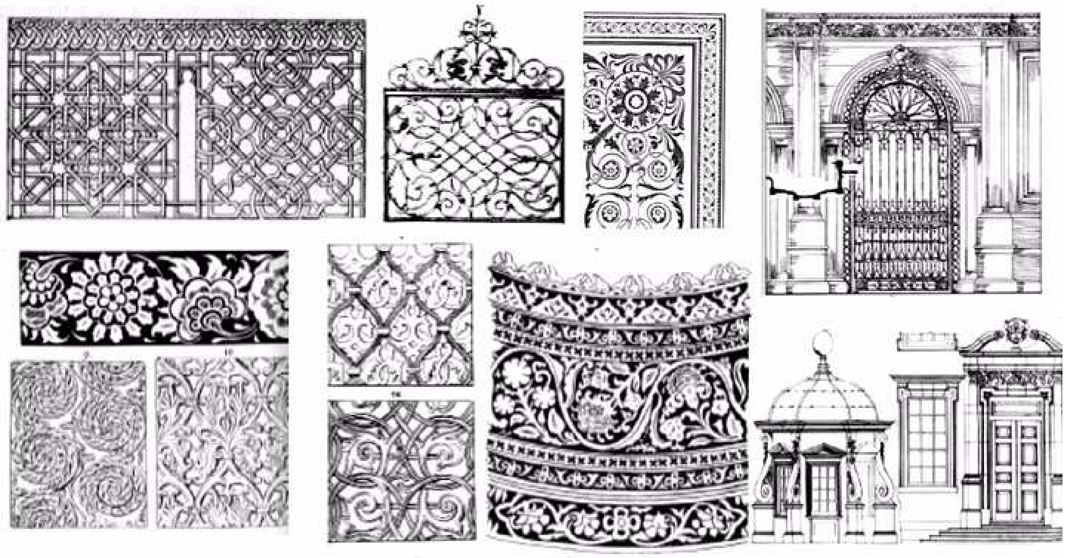

Fig. 15. A variety of ornamental patterns from various cultures and historical periods, showing the common principles of symmetry and periodicity in simple and compound hierarchical forms.

aesthetically pleasing when played together, are related by simple arithmetic or integer harmonies, as discovered already by Helmholtz in 1863.

Helmholtz’s theory of consonance was the first concrete evidence that at least one aesthetic function (in this case the perception of harmony between musical notes) is explained by a resonance phenomenon. Others have also noted the connection between aesthetics and harmonic resonance. Mary Waller noted how many Chladni figures have a distinctly ornamental character, examples are shown in Figure 16. Figure 16 A shows Mary Waller demonstrating her dry ice stimulation technique on a square steel plate, and (Figure 16 B) on a pentagonal plate. Figures 16 C and D show some of the many patterns obtained on triangular and square plates respectively. These resonances are as aesthetically pleasing as the finest ornamental wrought iron fence or arabesque tile pattern.

Another pioneer of aesthetic science was Ernst Häckel, whose book *Kunstformen der Natur*, (1904, the art forms of nature) highlighted the profoundly aesthetic and ornamental aspects of so many different life forms, most especially the simplest life forms at microscopic scale. We now know the foundational basis behind the periodicity and symmetry of these patterns is a chemical harmonic resonance. Figure shows two pages from Ernst Häckel, whose book *Kunstformen der Natur*. The reader is encouraged to see more at http://en.wikipedia.org/wiki/Kunstformen_der_Natur.

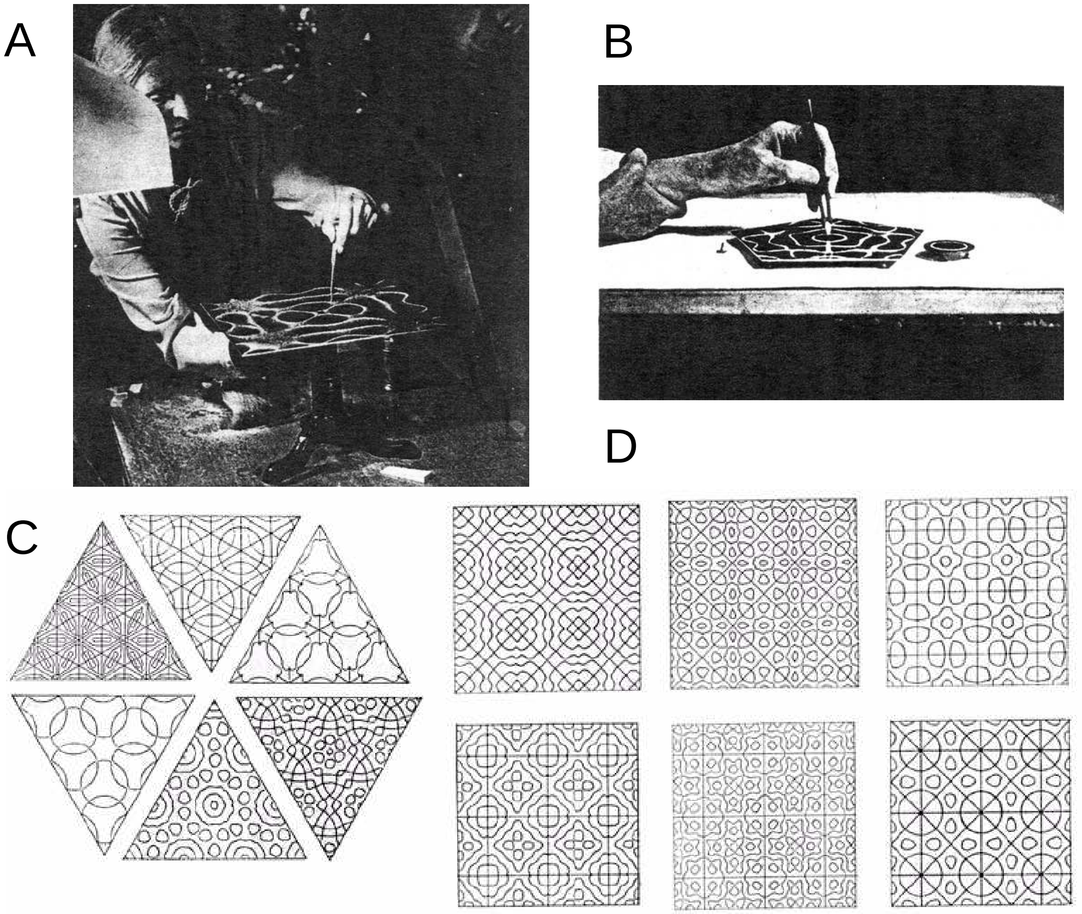

Fig. 16. A: Mary Waller creating an ornamental Chladni figure by pressing a piece of dry ice held in tweezers against the plate. B: A Chladnin figure on a pentagonal plate. C and D: Ornamental standing wave patterns on triangular and square plates respectively. Photographs and Chladni figures from Waller (1960).

The existence of human music is perhaps the most convincing evidence for a harmonic resonance theory of brain function. The same ratios that we find pleasing aesthetically, are the very ratios on which our number system is based, and the same kinds of ratios extended into three dimensions correspond to the architectural and sculptural proportions that we consider aesthetically to be pleasing and balanced and harmonious. The connection between music and human behavior can be seen by the effect that human music has on human behavior. Music tends to make us gyrate and oscillate in periodic and symmetrical motions in both simple and compound hierarchical patterns. It would take an alien visitor to notice those aspects of our nature that are so common and pervasive throughout every culture and throughout history, becase we humans have become so habituated to these prominent characteristics of human aesthetic preference that they are almost completely invisible to us. We simply take it for granted that periodic and symmetrical patterns are intrinsically beautiful. It needs

Fig. 17. Two of many pages from Ernst Häckel's breathtaking book *Kunstformen der Natur*. revealing a profoundly ornamental, and aesthetically pleasing aspect of simple life forms. We now know that the periodicity and symmetry of these body forms is itself a consequence of a chemical harmonic resonance.v

neither to be proven nor justified, it is just how things are. And we think nothing is strange when a person in our company suddenly gets up and starts gyrating wildly in compound oscillatory motions, with their face screwed into expressions of extreme agony or ecstacy, just because they hear patterned vibrations in the air. We just turn aside and sip our drink, or jump up and join them, as if this were the most normal behavior in the world. But in fact this wild profusion of symmetry and periodicity throughout human affairs is very telling of the foundational principles behind how our brains operate, just as the symmetries and periodicities in our bodies are telling of the mechanism of embryological morphogenesis.

The reason why beautiful music sets our body into motion is because music is the language of the brain. It is what integrates the millions of individual brain cells into a single unified resonance, resulting in a unified global experience in the form of spatial patterns in our brains. The individual neurons are doing very much what human dancers do when they all dance to the same tune. The cells all begin to oscillate electrically in a synchronous global wave that unites the individual activations into a larger global wavelike entity that emerges, as if out of nowhere, across neural tissue in beautiful periodic spatial patterns, paying only the scantest heed to the barrier of the cell wall. The lawful geometry of the standing wave patterns in the brain is very much like the patterns seen in visual ornament, with basic elements repeated in periodic and/or symmetric patterns of infinite variety, whose elements are defined by the more general symmetry of the line, the circular arc, the right angle, sinusoids, zig-zags, crenellations, and other lawful patterns of lines and curves. And the patterns in the brain are also similar to the patterns in mathematics and in music. It is a pattern that requires energy for its construction, and releases energy in its destruction. But it is a spatial structure that is composed of nothing but energy, magically superimposed on the physical tissue of the brain. This is the "ghost in the machine", the substance of *mind*, that peculiar, immaterial substance that pops spontaneously into existence in a waking brain, like a note from a horn when it is blown, and disappears again without a trace when we fall asleep, like a horn resting silently in a corner.

Universal Resonance

One of the most profound discoveries of modern science has been the fundamentally wave-like nature of matter, and the fact that matter and energy are mutually inter-convertable. The fundamental particles of matter, protons, neutrons, electrons, are now conceived of as standing waves, whose discrete nature is a manifestation of the discrete nature of resonance itself. Electrons are not point-like particles in orbit around the nucleus, but waves that occur only at those radii or in those configurations in which they interfere with themselves constructively. A particle is like light that is trapped in a feedback loop, going round and round, or back and forth, indefinitely about the same point. It is harmonic resonance that provides the structural patterns of matter. For example electron oribitals come in different modes, based on different standing wave functions, as shown in Figure 18. The spherical-symmetrical *S-mode* atomic orbitals define a series of concentric spherical shells, a radial in-and-out resonance that is isotropic, the same in all directions, as shown(in cross-section) in Figure 18 A. The sun and other stars breathe in and out radially in this manner, oscillating about an equilibrium level determined by the dynamic balance between the outward blast from the nuclear furnace at the core, and the weight of gas pressing inward. The *P-mode* orbitals define standing waves going back and forth across the center in one dimension, as shown in Figure 18 B, with compression in one direction balanced by expansion in the other. A baseball experiences this kind of oscillation immediately after contact with a bat, as the compression wave from the struck side propagates to the opposite pole and back, alternately compressing and expanding the two hemispheres in counterphase to each other. The higher

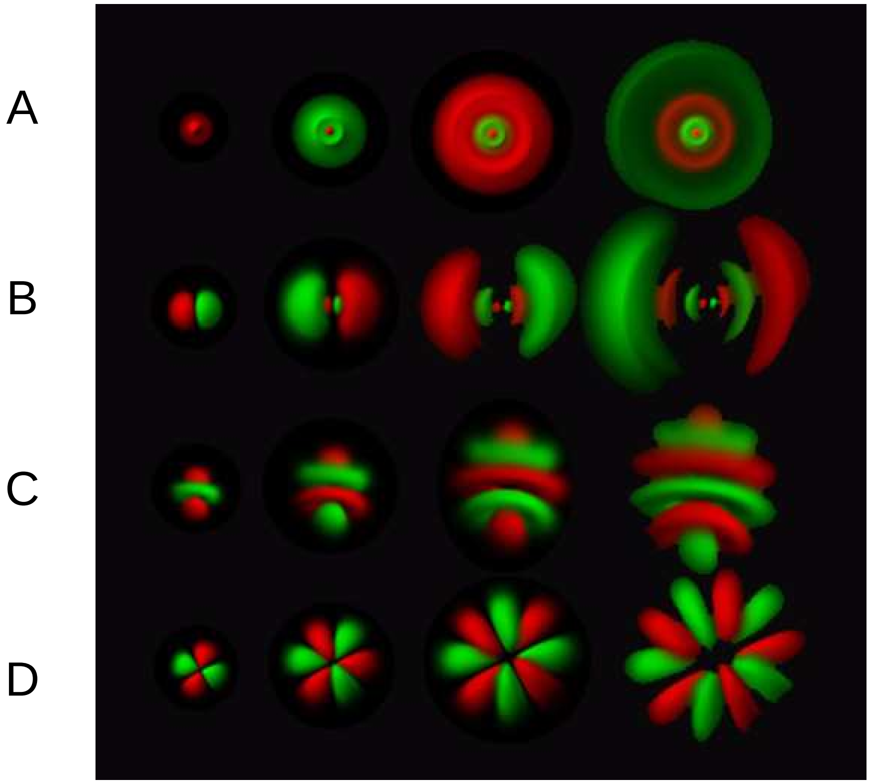

Fig. 18. Various modes of atomic orbitals. A: The s-mode is a concentric in-and-out vibration (cross section). B: The p-mode is a lateral vibration across one dimension. C: The d-mode orbital is equatorial/polar, like the lines of lattitude on a globe, and D: the f-mode is a circumferential or equatorial resonance, like the lines of longitude on a globe.

harmonics of this resonance add additional layers in periodic alternation. The *d-mode* orbitals shown in Figure 18 C represent an equatorial/polar oscillation, with motion from the equator towards the poles, alternating with motion from the poles back toward the equator. The higher harmonics of this oscillation create more alternating layers of reciprocating motions like the lines of lattitude on a globe. And the *f-mode* orbitals shown in Figure 18 D define a circumferential resonance running around the equator, creating an alternating pattern like the lines of longitude on a globe. Atomic orbitals are wave functions that are characterized by a perfect symmetry of positive and negative regions, periodically distributed across some dimension. These are the patterns taken on by electrons in orbit about a charged atomic nucleus, but they are also the patterns observed in falling droplets, or flying baseballs, or stars and planets, when they rebound and recoil after energetic collisions. They all exhibit a remarkable self-organizing tendency to spontaneously sub-divide the space of their possible motions into periodically and symmetrically arranged alternating volumes, with exactly balanced reciprocating motions.

Harmonic resonance is *the* preeminent organizational principle of matter and energy in the universe. Like life itself, harmonic resonance represents a rare example where the *entropy* or disorder of a system is not increased, but is actually reduced with the onset of resonance, a spontaneous emergence of ordered spatial pattern out of a high entropy homogeneous medium. Harmonic resonance is the force that binds waves into particles of matter, and it is the harmonic resonance of chemical bonds that lock particles into bulk material substance. The octet rule that makes atoms swap or share electrons in an attempt to form complete outer shells, is itself a resonance phenomenon. A planet spinning on its axis is a kind of round-and-round pendulum, a rotational resonance with the surrounding universe, as is the orbit of a satellite around its planet, which can range from a near circular orbit to an extreme back-and-forth pendulum motion. Harmonic resonance is the force that organizes planets in their orbits, and stars in their galaxies. And it is harmonic resonance that binds the individual activations of countless billions of individual neurons into the coherent integrated framework of experience and willful action that we call Mind.

Chapter 2

What Is Harmonic Resonance?

Harmonic resonance is an extraordinarily diverse and varied phenomenon seen in countless forms throughout the universe, from gravitational orbital resonances, to electromagnetic oscillations, to acoustical vibrations in solids, liquids, and gases, to laser resonance in light and microwaves. Harmonic resonance spans a vast range of spatial scales, from the tiniest wave-like vibrations of the elemental particles of matter, to orbital resonances that emerge from spinning disks of gas and stars. But across this vast range of spatial scales and diverse media, there are certain general properties of harmonic resonance that are common to all of them. They all tend to oscillate at some characteristic frequency, and at its higher harmonics, frequencies that are integer multiples of the fundamental frequency. They all exhibit spatial standing waves, whose wavelength is inversely proportional to their frequencies. They all tend to subdivide one, two, or three-dimensional spaces into equal intervals of alternating reciprocating forces dynamically balanced against each other, with the twin properties of periodicity and symmetry across every possible dimension of space and time. These, and many other properties, are properties of resonance in the abstract, manifested across all those diverse forms and media. Harmonic resonance is a higher order organizational principle of physical matter, that transcends any particular implementation in a physical medium. It is the properties of that transcendent, more general concept of harmonic resonance that are the focus of this book, because it is those transcendant properties that reveal the essential properties of resonance itself, and explain how those properties lead to the emergence of mind from brain.

The minimal prerequisite for harmonic resonance is some system that when deflected from some rest state, or equilibrium condition, experiences a restoring force that pushes it back toward that equilibrium state. Also required is some kind of inertia, or momentum term, that makes it overshoot the equilibrium point and pass on through, continuing on to a deflection of equal magnitude in the opposite direction, from which point the restoring force will accelerate the system back toward the equilibrium center again, setting up for repeating back and forth oscillations that can continue indefinitely in the absence of frictional losses.

The simplest harmonic resonances can be found in highly constrained dynamic systems, like a pendulum that is free to swing only within a plane, or a linear mass-and-spring system sliding back and forth on a frictionless surface. This kind of resonance is known as *simple harmonic motion*, and it has a number of beautifully harmonious aspects or symmetries. The position-time trace of a swinging pendulum or mass-and-spring system describes a sinusoid back and forth across an equilibrium point, with a constant and continuous reciprocal exchange between potential and kinetic energy. The sinusoid is circular motion in projection, constantly accelerating up and down at a rate that itself follows a sinusoidal function, an acceleration profile that is 90 degrees phase-advanced to the motion it induces. It is a perfectly regular curve that follows a simple law of acceleration with a harmonious dynamic geometry.

The equation for simple harmonic motion is given by

$$x(t) = A\sin(2\pi ft + \delta) \qquad \text{(EQ 1)}$$

where $x(t)$ is the displacement from the origin at time t, A is the amplitude of oscillation, f is the frequency, and δ is the phase of the oscillation. Differentiating once gives an expression for the velocity at any time.

$$v(t) = \frac{d}{dt}x(t) = A\omega\cos(\omega t + \delta) \qquad \text{(EQ 2)}$$

and differentiating again gives the acceleration at any time.

$$a(t) = \frac{d^2}{dt^2}x(t) = -A\omega^2\sin(\omega t + \delta) \qquad \text{(EQ 3)}$$

The mathematical characterization of simple harmonic motion as a sinusoidal oscillation that repeats exactly in each cycle, captures the constant unchanging aspect of harmonic resonance. But some of the most interesting aspects of resonance occur as a distortion of that endless pattern, as the resonance resists the distortion and tries to restore the symmetry of the perfect periodic pattern. In the phenomenon of entrainment, two oscillators, like pendulum clocks hung next to each other on a wall, will subtly distort each other’s oscillations a bit at a time, bending and warping each sinusoidal time trace until they are swinging in lock-step counterphase harmony.

The simple harmonic oscillator has another peculiarity that is of significance: it responds not only to periodic forces applied at its natural fundamental frequency, but it also responds to higher harmonics of that frequency. For example if a pendulum, initially motionless, is tapped periodically at exactly double its fundamental frequency at the moment it reaches the equilibrium point, it will begin to swing half-cycles, making repeated excursions in one direction only, to reverse abruptly at the equilibrium point as it rebounds off the next tap. Although it takes precise tapping at precisely the right time and strength to achieve this kind of motion, this peculiar property of the simple harmonic oscillator opens the possibility for setting up pairs of identical pendulums swinging against each other, colliding and rebounding off each other across the equilibrium point, creating a double oscillation of mirror-symmetric motions at double the fundamental frequency. In fact, any number of simple harmonic oscillators can be strung together in this manner to create compound oscillator systems. For example mass-and-spring oscillators can be chained together into a string of masses connected by springs, each one a simple harmonic oscillator, but together they form a complex compound oscillator that exhibits many more levels of resonance than the simple harmonic components of which it is composed.

Lissajous Figures

Simple harmonic motion gets a lot more interesting when you allow it a second dimension of freedom. For example a pendulum that is free to swing in both x and y dimensions can describe all kinds of complex elliptical orbits that continuously exchange potential and kinetic energy across both x and y dimensions. A similar two-dimensional oscillation is seen in the *Lissajous curves* on an oscilloscope, achieved by plotting two sinusoidal oscillations against each other, one in x and the other in y, as shown in Figure 2.1. The first row in Figure 2.1 shows two sinusoidal oscillations of the same frequency plotted against each other, with a range of phase shifts between the two oscillations along the columns from 0 to π in increments of $\pi/8$. At zero phase difference the plot oscillates back and forth along the $y = x$ diagonal line. At a phase shift of $\pi/2$ (or 90 degrees) the plot oscillates round and round a perfect circle, and at a phase shift of π (180 degrees) the plot forms a diagonal line tilted the other way, along the $y = (-x)$ line. Subsequent rows in Figure 2.1 plot two sinusoids of different frequencies, where the frequency of the x oscillation is either an integer multiple (2, 3, 4) or a rational fraction (1/2, 3/2, 4/3) of the frequency of y.

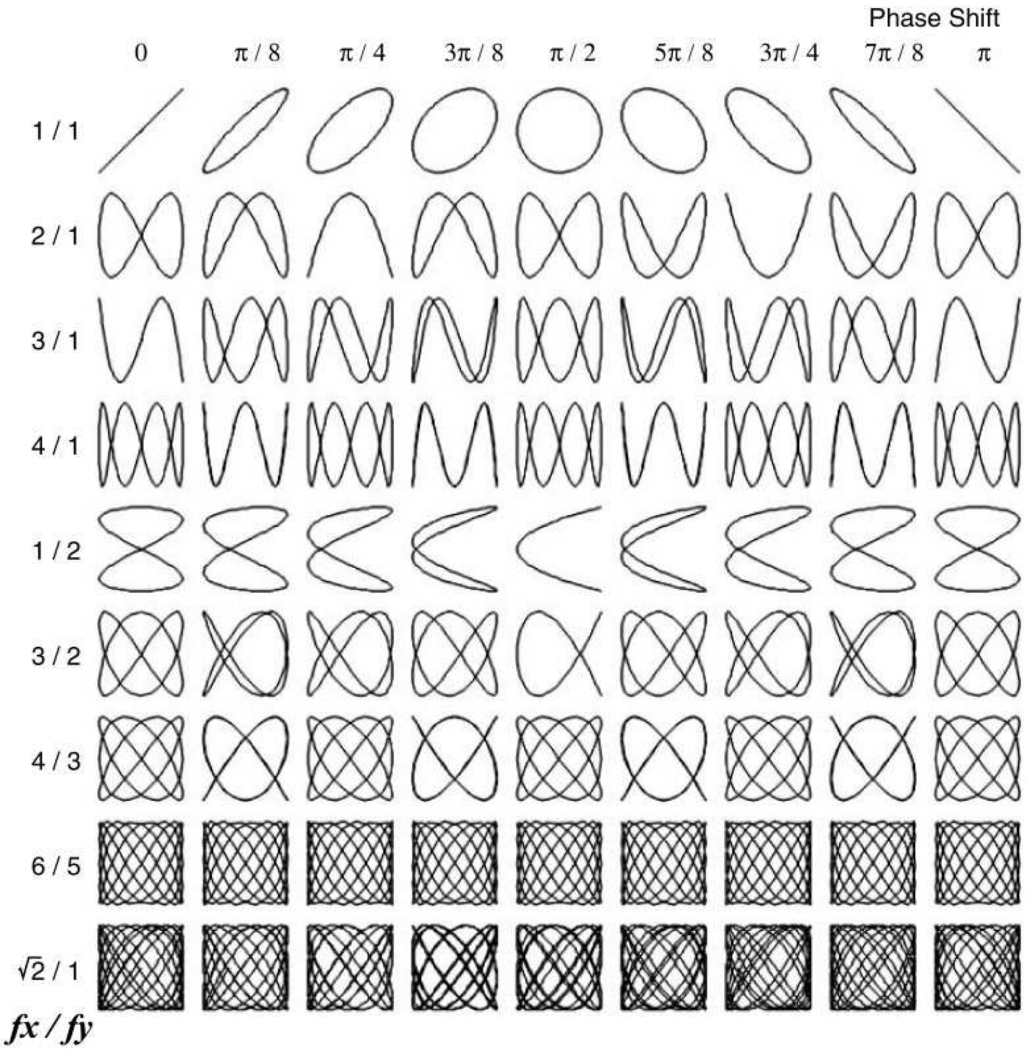

Fig. 2.1. Lissajous figures, created by plotting one sinusoid in *x* against another in *y*. The columns show various phase shifts between the two oscillations, in increments of π/8. The rows show the effect of varying the frequency of x relative to that of y in integer ratios, to create closed figures. The last row shows an irrational ratio that defines an open figure that covers new ground for ever until the whole plot turns black.

If the frequencies of the x and y oscillations are matched, with a 90 degree phase lag between them, the Lissajous figure forms a circle. If the frequency of one is exactly double that of the other, it produces a figure 8 shape. Any other integer relation between the two frequencies produces other closed lissajous curves, like those in Figure. However only harmonically related frequencies form closed trajectories, or static Lissajous figures. If the frequency f_x is not an integer multiple or fraction of f_y then the pattern will cycle endlessly across the scope, a travelling wave rather than a standing wave, never quite retracing exactly the same pattern. This is shown for a partial trace in the last row of Figure 2.1, that plots a frequency ratio of the irrational fraction square root of two. The plot *almost* retraces its path each cycle of the oscillation, but not quite, and if allowed to run forever, the plot would eventually fill the figure entirely with a solid black field when the trace lines get close enough to abut each other, although mathematically it would always be covering new ground at ever finer scale. Once again we see a very simple system composed of two independent oscillators, that produces a fantastically complex repetoire of beautiful periodic and/or symmetrical patterns when combined.

Lissajous figures can also be produced in three dimensions by plotting sinusoidal oscillations in *x, y*, and *z* dimensions, some of which are shown in Figure 2.2, where they are displayed in a grid sorted by their frequencies in the *x*, *y*, and *z* dimensions.. Again, only integer ratio frequencies are shown, because they are

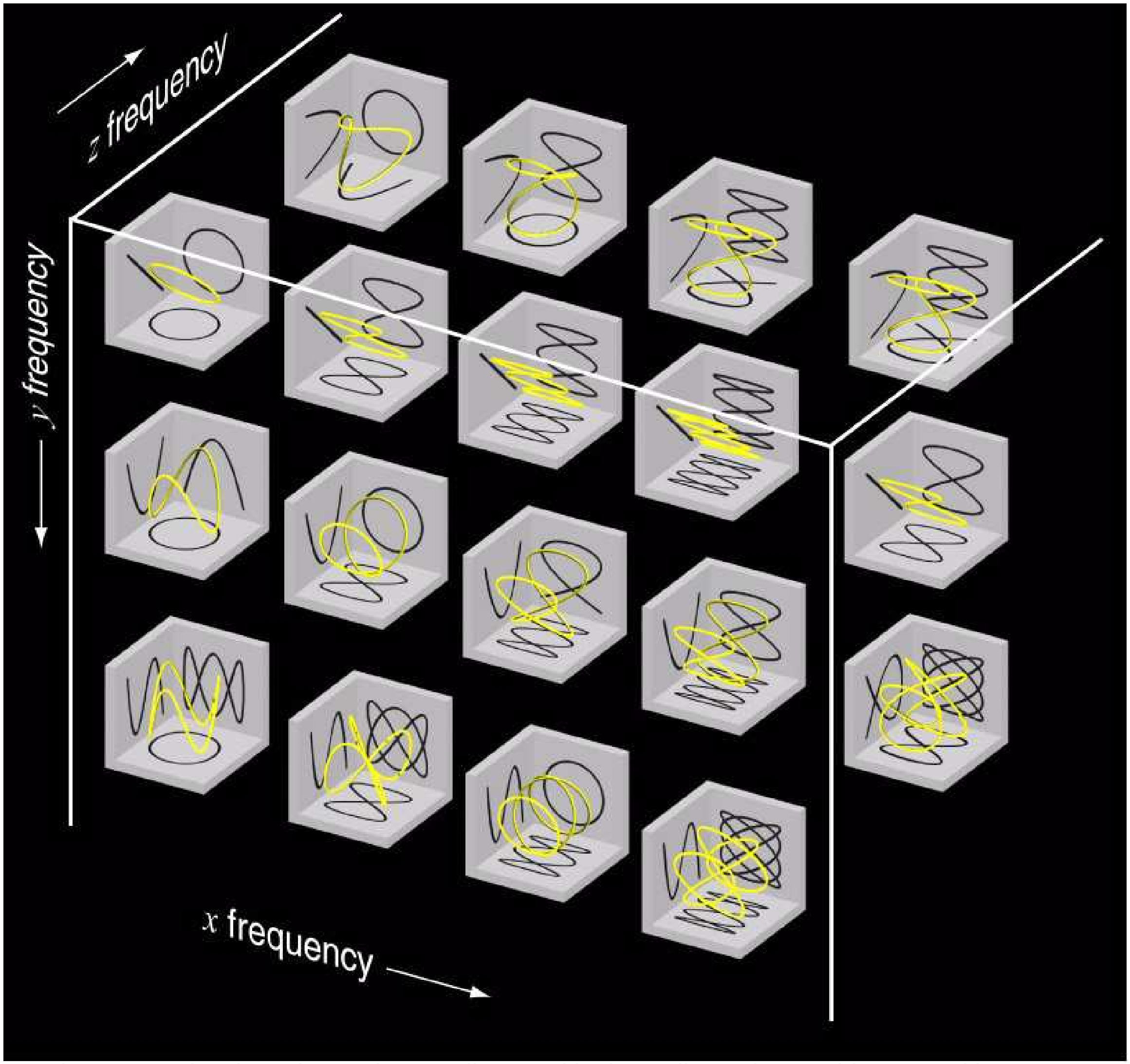

Figure 2.2. Lissajous figures in 3-D, displayed in three-dimensional grid arranged by *x*, *y, z* frequencies in the ranges *x* {1-4}, *y* {1-4} and *z* {1-2}. The phase of *y* is arbitrarily advanced by $\pi/2$ relative to *x*, while the phase of *z* is retarded by $\pi/2$. Other phase ratios produce even more arrays of closed Lissajous figures than those shown here.

the only ones that produce closed figures. Also, in this figure only one relative phase relation is shown between the *x*, *y*, and *z* waveforms, (arbitrarily, the phases of y and z are advanced and retarded by π/2 relative to that of x, respectively). Other phase ratios (in rational fractions of 2π, as in Figure 2.1) produce still more arrays of closed patterns beyond those shown in this figure. Again, we have a very simple system of three oscillations that together define an enormously complex array of periodic and symmetrical patterns in a lawfully organized hierarchy. There are always many more irregular, or open figures in between the symmetrical closed figures shown in Figure 2.1.

Physically, these same kinds of patterns can be achieved quite simply by suspending a mass on springs, as suggested in Figure 2.3 A, and twanging it like a guitar string to set it into vibration. This creates a compound oscillator whose natural modes of vibration correspond to the family of three-dimensional Lissajous figures. Actually, a single mass-and-springs system will naturally oscillate not only at the discrete integer ratio frequencies of the closed Lissajous figures, but also at any of the many intermediate irregular frequency ratios of the travelling wave Lissajous figures, although more complex compound systems can possess emergent interactions that preferentially promote the harmonic oscillations of the Lissajous figures, as explained below.

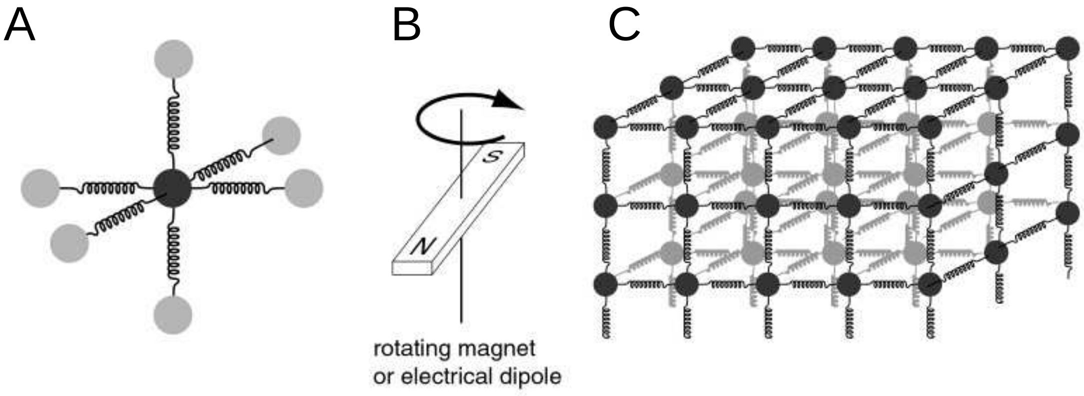

Fig. 2.3. A: A mass suspended on springs will naturally tend to oscillate in three-dimensional Lissajous figures. B: If a bar magnet, or electrostatic dipole, is rotated in the viscinity of a sprung mass that is magnetized, or electrically charged, respectively, it will set the mass into vibration at the right rotational frequency. C: A block of sprung masses of that sort will also respond to a rotating magnet or dipole either in random, thermal vibration, or in coherent oscillations, by the same principle that electromagnetic radiation is absorbed by solids, when its frequency matches the natural modes of oscillation of the system.

Molecular Vibrations

Atoms that are locked in solids each act as simple harmonic oscillators in three dimensions, much like a mass held in place by springs, as depicted in Figure 3 A, as each atom in the glass or crystal lattice is held in its place by elastic forces of attraction and repulsion. If an atom in a solid is “twanged”, or knocked out of its central equilibrium position, it will automatically follow an oscillatory path that traces out a three-dimensional Lissajous figure, like a mass-and-springs system.

The electrostatic forces that hold atoms in their molecular places, act over a quite limited range. But atoms also interact across much longer distances by the principle of entrainment and resonance. An agitated atom in a lattice vibrates energetically at its natural resonant frequency, and that tiny oscillating electrostatic charge radiates energy outward in all directions as an electromagnetic field, that appears in the form of a miniscule, but constantly reciprocating force of modulated attraction or repulsion. That tiny alternating force can be detected by a distant atom that happens to share exactly the same resonant frequency, because the tiny pushes of the alternating field accumulate over time, as when pushing a child on a swing with carefully timed shoves, and thus the effect of that reciprocating field is magnified by orders of magnitude in influence, compared to the static force of attraction or repulsion, whose effect is negligable except at very short range. This principle can be demonstrated by analogy, by making the sprung mass of Figure 2.3 A out of a magnet, or charged mass, and setting up a bar magnet, or electrostatic dipole, in the viscinity of that sprung mass, mounted on a pivot so that it can be rotated about its axis, as suggested in Figure 2.3 B. Although the position of the sprung mass is barely influenced by the rotation of the bar magnet to different orientations, if the bar magnet is rotated continuously at exactly the resonant frequency of the sprung mass, it will set the sprung mass into vibration. This is the principle behind the emission and absorption of electromagnetic radiation by atoms and molecules of matter, which is itself a resonance effect. The different properties or effects of the various bands of electromagnetic radiation on physical matter, such as gamma rays, X-rays, ultra-violet, visible light, infra-red, microwaves, and radio waves, is due to the fact that different frequencies resonate with different components or groupings of material substance. Gamma rays and x-rays are the highest frequency radiation, with enough photon energy to strip the electrons off atoms altogether, ionizing the absorbing matter into positive nuclei and free electrons, with destructive consequences to fragile molecular structures like biological tissue. Ultra-violet radiation is absorbed by the lowest level electron shells, nearest to the atomic nucleus, which also tends to ionize the atoms and thus knock atoms out of their places, whereas visible light is aborbed chiefly by the higher level electrons, raising them to higher energy levels, from whence they spontaneously decay again, releasing another photon by re-emission, without disrupting the underlying molecular structures. Infra-red radiation is absorbed by whole molecules, causing them to oscillate as thermal vibrations, or heat. Microwave radiation causes molecules to rotate or twist in place in their lattice locations. Radio waves are so low in frequency that they generally pass through bulk matter without interacting with it, which is why you can listen to your radio inside your house, since the waves pass transparently through the walls of your house. But radio waves are absorbed by metals, like the antenna of your radio, and the reason for this is also an interesting resonance effect.

The electrons in a non-metal are tightly bound to their host atoms, and thereby locked into the glass or crystal lattice. This is also true for most of the electrons of a metal. But metals, by their nature, have extra electrons in their partially-filled outermost electron shells that are not so tightly bound to their host atoms, but can wander about freely through the bulk material of the metal. These free electrons flow between the atoms in the lattice as a continuous “sea” of electrons, that behaves much like a fluid, although that negatively charged electrical fluid remains trapped strictly within the confines of the bulk of the metal solid, because if an electron were to escape, the bulk matter would instantly become positively charged, and that positive charge would instantly suck the errant electron (or another one like it) back into the metal solid to restore the electrical balance. This “sea” of electrons thus behaves like an emergent larger object, an elastic object on the size scale of the bulk material itself. The resonant frequency of this “sea” of electrons, therefore, is not a property of the individual electrons of which it is composed, but rather, it is a property of the larger global entity into which they are seamlessly merged. The entire antenna, in effect, behaves like a single electrial entity, with a resonant frequency that is a function of the size and shape of the whole antenna, rather than of its component atoms or electrons. That is why antennas have to be carefully tuned to match the frequency of the radiation that they are designed to receive, or to transmit, with longer antennas used for long wave radio, and progressively shorter antennas for short wave, VHF (very high frequency), and UHF (ultra high frequency) radio waves.

The principle of resonances of bulk matter can also be demonstrated by analogy, by setting up an array of masses connected by springs, as suggested in Figure

2.3 C. Although each mass has its own resonant frequency, as in Figure 2.3 A, the masses are not independent, but intercoupled by the array of springs. If a springy array of masses of this sort is “twanged”, it will tend to wobble like a jelly, as the vibration is communicated from atom to atom throughout the bulk matter. The oscillatory behavior of such a system depends not so much on the forces on its individual atoms, but more on the bulk properties of the array as a whole, in particular, its size and shape. This is emergence of yet another form, the resonant properties of the whole being far more than a simple sum of its component parts. The selective amplification of harmonic oscillations over chaotic or inharmonious ones stems from the fact that the chaotic, random oscillations tend to cancel each other out, whereas any coherence in the oscillations of groups of atoms will feed back on itself each cycle, which eventually produces global coherent oscillations from the energy of initially random noise. Crystals of solid matter also vibrate in this holistic manner, with coherent oscillations coursing back and forth across the crystal as a whole, at a frequency that is determined by the bulk geometry of the crystal. This is the principle behind the crystal oscillators that are used in electronic watches, and to time the data cycles in digital computers.

A crystal oscillator is a tiny piece of crystal, often quartz, mounted between electrical plates. The crystal is set into electro-mechanical oscillation by a randomly alternating electrical voltage across the plates, by the piezoelectric effect, whereby an electrical voltage across the crystal causes a physical deformation of the crystal, and conversely, the physical deformation of the crystal causes an electrical voltage. This is the principle by which sparks are generated when knocking two pieces of quartz together. If a random-noise alternating voltage is applied across the crystal, it will set the crystal into electromechanical vibration at its resonant frequency, which in turn generates a periodic electrical voltage, similar in principle to the acoustical tone produced when blowing a stream of air across the mouth of a bottle, and to the note produced by a violin string by the random rasping of a rosined bow, and the tone produced in a bugle by blowing a rude “raspberry” into the mouthpiece, and the beautiful harmonic patterns that appear on a Chladni plate when pressing a piece of dry ice against it. In fact, the resonances of a crystal oscillator of this sort are much like the modes of a *harmonic resonance continuum*.

Harmonic Resonance Continuum: The Wave Equation

The most spectacular emergence of harmonic resonance is observed when provided with the extra dimension of variability of a *spatial continuum* of vibrating substance. For example if one end of a rope is shaken vigorously with a periodic oscillation, travelling waves propagate away from the point of shaking, each point on the rope oscillating back and forth across the average location of the rope, or equilibrium line, like a *continuous array* of simple harmonic oscillators. Adjacent points on the rope are flexibly connected so as to impose a smoothness, or continuity constraint along the rope as a whole, forming alternating waves of deflection in the rope that can run continuously like travelling waves, or can reflect off an attachment point at the far end of the rope, and thus produce standing waves. Similar waves can be produced in a slinky, by waving it up and down, which produces transverse waves, as in the rope. But the longitudinal elasticity of the slinky also allows for longitudinal waves of compression and rarefaction, stimulated by pumping the slinky back and forth in a direction parallel to its length. This is directly analogous to the pulses of compressed and rarefied air that constitute sound waves. As with the rope, the slinky can produce either travelling waves or standing waves, depending on whether the far end of the slinky is free, allowing the travelling waves to just run right off the end, or fixed to a rigid support that reflects the waves back in the opposite direction.

The propagation of waves through a continuous elastic inertial medium of this sort is modeled mathematically by the *wave equation*, developed by D’Alembert, and refined by Euler. This equation can be derived by considering the physics of a chain of masses connected by springs, using Newton’s laws of motion and Hooke’s law for linear springs. The back-and-forth alternating motion of each mass reveals a cyclic and reciprocating pattern of acceleration. The forces that produce this acceleration are described by Newton’s second law, that is

$$F_N = ma(t) = m\frac{\partial^2}{\partial t^2}u(x, t) \tag{EQ 4}$$

where F_N is the horizontal force due to Newton’s law, *m* is the mass, and *a(t)* is the acceleration at time *t*, which can be expressed as a function of the second partial derivative of displacement *u* with respect to *x*, where *u* is expressed relative to its equilibrium location. The force that is causing this acceleration is the force due to the tension in the springs that connect the masses, which varies with the distance between adjacent masses by Hooke’s Law, that is,

$$F_H = F_{x-h} + F_{x+h} = k[u(x-h, t) - u(x, t)] + K[u(x+h, t) - u(x, t)] \tag{EQ 5}$$

where F_H is the total spring force on the mass, which is the sum of the spring forces from the two adjacent neighbors, which in turn is equal to spring constant k times the difference in displacement between the central mass and its two adjacent neighbors. This allows the following equation of force as mass times acceleration.

$$m\frac{\partial^2}{\partial t^2}u(x, t) = k[u(x-h, t) - u(x, t) + u(x+h, t) - u(x, t)] \qquad \text{(EQ 6)}$$

If the array of masses consists of N masses spaced evenly over the length $L = Nh$ of total mass $M = Nm$, and the total stiffness of the array $K = k/N$ we can rewrite the above equation as:

$$\frac{\partial^2}{\partial t^2}u(x, t) = \frac{KL^2}{M} \cdot \frac{[u(x-h, t) - 2u(x, t) + u(x+h, t)]}{h^2} \qquad \text{(EQ 7)}$$

We can now convert this problem from the discrete case, with N distinct masses separted by a distance h between masses, to a continuous case more like the equation for a slinky, where the mass and the spring force are distributed uniformly across the length of the slinky spring, that is, in the limit as $N \rightarrow \infty$, and $h \rightarrow 0$, we have

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} \qquad \text{(EQ 8)}$$

where u is the degree of deviation from the equilibrium location, x is the distance along the linear medium, and c is the speed of wave propagation through the medium. If the displacement u is defined as a vertical displacement, at right angles to the direction of wave propagation, the wave equation applies to transverse waves, whereas when u is defined as a longitudinal displacement parallel to the direction of wave propagation, the same equation applies to longitudinal waves. In fact, the wave equation applies to a vast range of resonating systems, from mechanical vibrations, to electromagnetic oscillations of light, radio waves, microwaves, to sound waves, and a slightly modified form of the wave equation is seen in the Schrödinger equation that defines the quantum resonances of atoms, electrons, and sub-atomic particles. The wave equation captures the very essence of the higher order organizational principle behind harmonic resonance that is common to all of the diverse physical manifestations of resonance.

Let us examine the meaning of the wave equation, using the example of air pressure waves in a linear tube, like those in a flute when a note is blown. Figure 2.4 A and B depict the instantaneous air pressure in a tube, open at both ends, at the two extremes of the cycle of a first harmonic oscillation. In Figure 2.4 A, there is an over-pressure at the center of the tube, indicated by the darker shading, and an under-pressure at the two ends, while in Figure 2.4 B there is an under-pressure at the center, and over-pressure at the two ends. Figure 2.4 C and D show a plot of *u(x)* as a pressure or density function along the length of the tube. Figure 2.4 E and F show the partial derivative of this pressure function with respect to *x*, which is the *gradient* of the air pressure, positive where the gradient of *u(x)* is rising with distance *x* along the tube, and negative where it is falling. Note how the gradient *du/dx* is zero at both the peaks and the troughs of the air pressure function *u(x)*. Figure 2.4 G and H show the double partial derivative d^2u/dx^2, that is, the *gradient of the gradient* of air pressure along the tube, or how steeply the gradient is rising or falling at each point along the tube. .

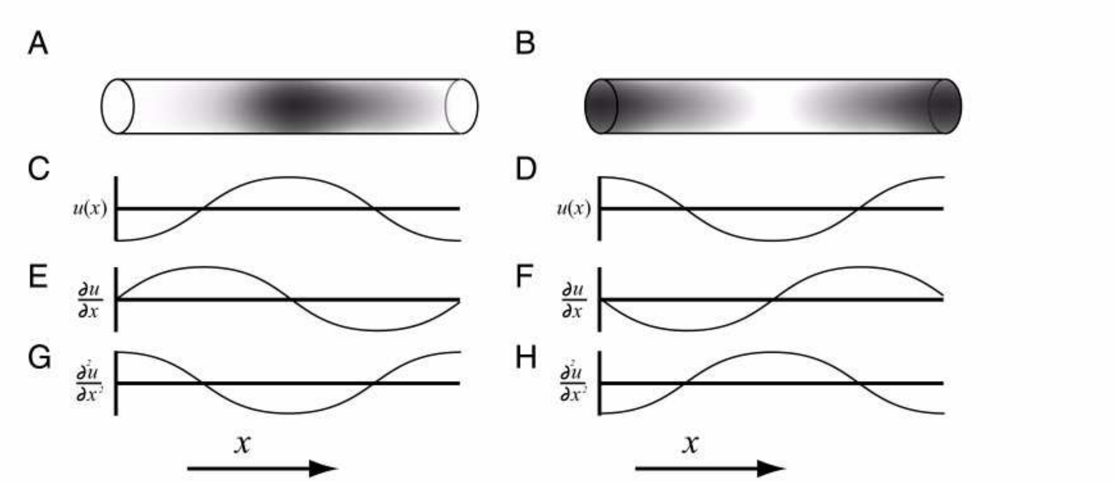

Fig. 2.4. A and B: Instantaneous air pressure function during the two extreme points in the cycle of a first harmonic standing wave in a tube, separated by 180 degrees in phase. C and D: Air pressure function expressed as *u*(*x*). E and F: The partial spatial derivative of the pressure function. G and H: The double partial derivative of the pressure function. This is the term that equates to the double temporal derivative of pressure in the wave equation.

Now the meaning of the wave equation can be stated in words as follows: The *rate of growth of the rate of growth*, or *accelerative increase* of air pressure at any point in the tube over time, ($\delta^2 u/\delta t^2$), is proportional to the *gradient of the gradient* of air pressure with distance along the tube ($\delta^2 u/\delta x^2$). In Figure 2.4 A the air pressure is high at the center of the tube, but the double derivative of air pressure is high at the ends of the tube. This means that the air pressure will be increasing acceleratively over time at the ends of the tube, and decreasing at the center, whereas in Figure 2.4 B where the pressure is high at the ends, the pressure will be increasing acceleratively over time at the center, and falling at the ends. This equation reveals harmonic resonance to be a *balance of accelerations in space against accelerations in time*. This deserves a little explanation. In a static air system air pressure $u(x)$ tends to distribute itself uniformly through the tube producing equal pressure throughout, as when a flute is lying quietly on a table. In a dynamic system with constant flow, (no acceleration) a gradient of air pressure, du/dx, will tend to establish itself along the length of the tube, for example when air is flowing along the tube due to a pressure difference between the two ends, as when blowing into a flute without making a sound. But the variations in air pressure due to harmonic resonance are a function of the *acceleration* of the air in the tube, ($\delta^2 u/\delta t^2$). The transient and alternating patterns of high and low air pressure in the tube are sustained by synchronously oscillating symmetrically opposed patterns of accelerations of air along the tube. For example the transient and momentary peak of positive pressure depicted in Figure 2.5 A, is caused by a decelerating collision between moving slugs of air arriving at that point from opposite directions, followed by an immediate rebound, shown in Figure 2.5 B, where those slugs of air recoil in opposite directions, creating a transient and momentary trough of negative pressure at that point and peaks at the ends. This in turn sucks the air back in toward the center, setting up for repeating cycles of alternating high and low pressure disposed as a spatiotemporal pattern of standing waves, periodic in space and time. The spatial continuity and uniform momentum and elasticity of the vibrating medium distribute the patterns of acceleration across continuous sinusoidally varying patterns of acceleration of the medium. This is the origin of the balance and symmetry properties of harmonic resonance. If the colliding and rebounding slugs of air were not perfectly and symmetrically balanced, for example if the slug on one side were slightly more massive than the one on the other side, then that would instantly and immediately relocate the collision point to occur where each slug has completely exhausted its momentum against the other, where the slugs of air are again balanced. This is the essential pattern formation principle behind standing waves of harmonic resonance, that is responsible for the regularity or order of the emergent pattern.

The laws of physics, in particular, Newton’s second law of motion, and Hooke’s law of elastic springs, dictate that the waveforms depicted in Figure 5 conform to the wave equation, that is, those functions are a “solution” to the wave equation, because the equation remains balanced or “equal” throughout the oscillations. There are other solutions to the wave equations besides the first harmonic. A second harmonic oscillation, with two peaks of over-pressure, alternating with two troughs of under-pressure, is another solution to the wave equation. The gradients in space and time are double those of the first harmonic, resulting in double the acceleration due to those pressure gradients, and thus also double the oscillation frequency compared to the fundamental. Other solutions are found in the third, fourth, and higher harmonics at correspondingly higher frequencies of oscillation.

Even this does not exhaust the repetoire of a harmonic resonance system, because it is not only the higher harmonics which provide solutions to the wave equation, but also combinations of harmonics, or compound waveforms that are a sum of a number of harmonic components, which also provide solutions to the wave equation. Given the fact that Fourier theory shows that virtually any waveform can be expressed to arbitrary precision as a sum of component sinusoids, we see that virtually any waveform can be expressed in the form of standing waves in a harmonic resonance representation, and those patterns that have the properties of symmetry and periodicity can be most easily expressed using only the lower harmonics that require the least energy. This is the reason why symmetry and periodicity form the foundations of aesthetic patterns in visual ornament, as well as in the patterns of melody and harmony and rhythm in music. Harmonic resonance is a *multipotential* pattern formation principle, that is, the same mechanism that generates the fundamental resonance can also create a range of patterns defined by higher harmonics and combinations of higher harmonics by the same essential principle.

Wave Equation in Two Dimensions

The wave equation extends naturally into two-dimensions, where it appears in the form

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u \qquad \text{(EQ 9)}$$

where ∇^2 is the *Laplacian*, which is defined as the sum of the unmixed second partial derivatives across *x* and *y*, that is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad \text{(EQ 10)}$$

Waves on the surface of water provide a two-dimensional spatial continuum of oscilating substance (water) with mass and inertia at every point in the surface, and a gravity-buoyancy restoring force directed towards an equilibrium level, as the water seeks to find a single level throughout the vessel.

The meaning of the wave equation in two dimensions can be expressed in the case of a water surface as follows: The rate at which the level of the water is accelerating upwards (or downwards) at any point in the surface is proportional to the gradient of the gradient of the water surface through that point, summed through x and y dimensions. To get an intuitive feel for the meaning of this equation we can go back to Figure 2.4 again, but this time consider the plot of $u(x)$ in Figure 2.4 C and D as a plot of the vertical displacement of the water surface in the x dimension, or a picture of the actual water surface waves (ignoring the y dimension). The center of the plot in Figure 2.4 C shows the peak of a water wave, while the plot of Figure 2.4 D is centered on a wave trough. In both cases, the gradient at the center du/dx is zero, as seen in Figure 2.5 E and F. But the gradient of the gradient d^2u/dx^2 of Figure 2.4 C, shown in Figure 2.4 G, is negative, because the wave is bowed upward, surrounded by lower water, and thus the vertical acceleration at that point is negative, i.e. the water level $u(x)$ is about to start dropping, while the trough of the wave in Figure 2.4 B is dished downward, surrounded by higher water, and thus the vertical acceleration at that point is positive, it is about to start rising, as shown in Figure 2.4 H. This gradient of the gradient is calculated independently in orthogonal x and y dimensions at each point in the surface, and the resultant vertical acceleration is computed as the sum of these unmixed second partial derivatives.

The Chladni plate also demonstrates the vibration of a two-dimensional spatial continuum, this time the substance being the steel plate, whose elastic flexibility serves as the restoring force. The Chladni figures demonstrate the true potential of a standing wave representation, with its hierarchically ordered families of patterns, all sorted into rows and columns of increasing vibrational energy. This is emergence in a most impressive form, the emergence of complex spatial structure from a simple unstructured homogeneous medium.

Wave Equation in Three Dimensions

Nature offers three dimensions of freedom for harmonic resonance, thus even more spectacular examples of standing waves can be obtained in three dimensions. The wave equation extends naturally into three dimensions in the same form as it appears in two dimensions as in equation 9, except in this case the Laplacian ∇^2 is defined as the sum of the unmixed second partial derivatives across the three dimensions of *x*, *y*, and *z*, that is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad \text{(EQ 11)}$$

Figure 2.6 shows a series of standing waves of acoustical resonance in a three-dimensional box or cavity. The green and red shades in this figure represent points of higher or lower instantaneous air pressure at each point in the cavity, depicted at an instant of maximal pressure excursion from the equilibrium condition. That is, the regions of high and low pressure reverse with each half cycle of the oscillation, the high pressure regions becoming low, and the low becoming high. The full cyclic oscillation of these volumetric patterns can be seen most clearly in Paul Falstad's excellent *Math and Physics Applets* http://www.falstad.com/, in particular, the *Box Modes Applet* http://www.falstad.com/modebox/ . Figure 6 was composed of static frames from Falstad's Box Modes applet. The reader is encouraged to check out this simulation on-line, and to play around with the applet to experience its full dynamic time-variant properties.

The standing wave patterns in Figure 2.5 are arranged in order of their harmonics in the *x* and *y* directions. The box in the upper left corner of Figure 2.4, or box [*x,y*] = [0,0], represents the zero$^{\text{th}}$ harmonic, like the DC term in a Fourier representation. The rest of the top row in Figure 2.5, from [1,0] through [3,0], represents various harmonics of oscillation in the *x* dimension. The first harmonic [1,0], has a high/low pressure profile, (alternating over time with a low/high profile) in the *x* dimension, and it oscillates at a fundamental vibration frequency that depends on the speed of sound through the medium. The second harmonic [2,0] has a high/low/high pressure profile (alternating with low/high/low) in the x dimension, oscillating at three halves (3/2) of the fundamental frequency; the third harmonic [3,0] has a high/low/high/low profile at twice the fundamental frequency, and so forth, to ever higher harmonics in x (not shown).

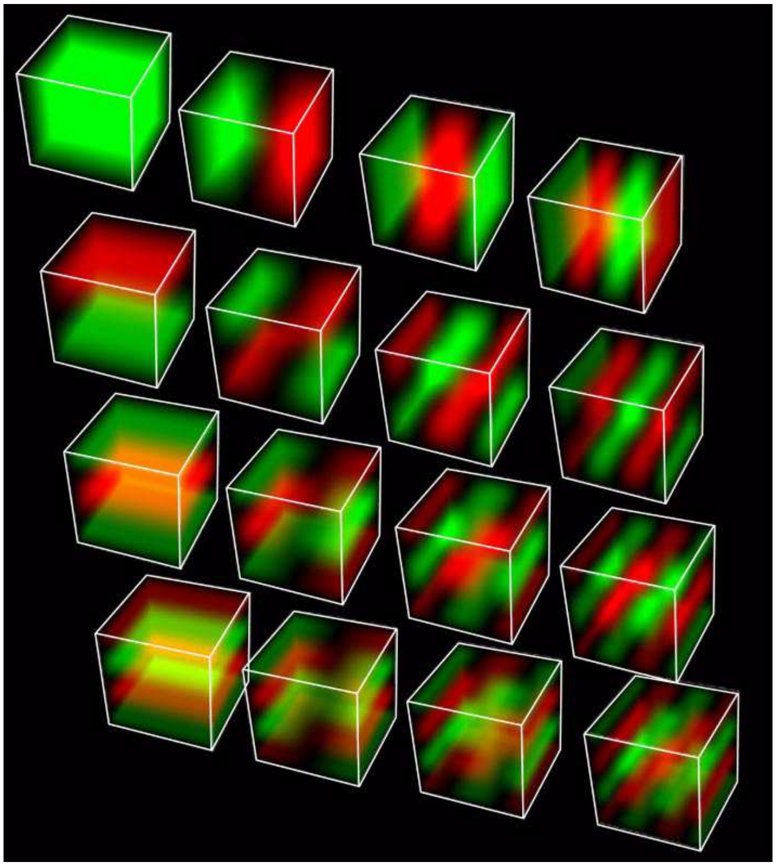

Fig. 2.5. Box modes applet from Falstad's Math and Physics Applets, showing standing waves in a cubical box, for example sound waves in an air-filled box. A rising sequence of harmonics in the x dimension is shown from column 1 through 4, and a rising sequence of harmonics in the y dimension in rows from 1 through 4. A third dimension (not shown) would coplete the figure as a 4 x 4 x 4 array of the harmonics in *x, y,* and *z*.

The left hand column in Figure 2.4, from [x, y] = [0,1] through [0,3] shows the first three harmonics of oscillation in the y dimension, from [0,1] high/low, through [0,3] high/low/high/low, this time in the vertical instead of the horizontal direction. The rest of the boxes in the array show the standing waves composed of both horizontal and vertical components. For example box [1,1] shows the first harmonic in *x* and *y* simultaneously. This defines a first harmonic of diagonal resonance in the [*x,y*] direction, whereas box [1,2] represents a first harmonic in *x*, and a second harmonic in *y*, and so forth. Each one of the standing waves in the array of Figure 2.5 depicts a separate and discrete pattern of standing wave oscillation that can be sustained in the box as a natural resonance.

Figure 2.5 shows only the standing waves in horizontal (*x*) and vertical (*y*) dimensions. Not shown are the standing waves in the depth dimension (*z*) into the plane of the page. To show all the standing waves of a cubical box up to the third harmonic would require expanding the two-dimensional 4 x 4 array of boxes of

Figure 2.5 into a three-dimensional 4 x 4 x 4 cubical array of boxes, with a separate and distinct standing wave pattern for every combination and permutation of [*x, y, z*] harmonics, an extraordinary repetoire of organized spatial patterns from a simple homogeneous resonating box. And all these geometrical organized templates, or patterns, can be called up either singly or in combinations, to produce a vast array of possible patterns, just by playing the right tone, or combinations of tones in a musical chord, in the presence of the box.

For a more concrete intuitive understanding of the principles behind these standing wave patterns, I will describe a simple physical system that could be used to produce volumetric acoustical standing waves as in Figure 2.5. We begin with a cubical air-filled box, or cavity, with three loudspeakers mounted on three orthogonal walls of the box, as shown in Figure 2.6. Each of these three speakers is driven by a signal generator that generates a sinusoidal waveform whose frequency and amplitude can be adjusted independently. Suppose we turn on only the signal generator for the *x* dimension, set at a very low frequency, and gradually sweep through a range of frequencies continuously from low to high. Wherever the frequency happens to match the fundamental vibration frequency of the box, or one of its higher harmonics, a powerful resonance will be heard in the box, as the standing wave amplifies itself by positive feedback. This will occur at discrete frequencies corresponding to the resonant modes of the box, as shown across the top row of Figure 2.5. Similar resonances can be obtained independently in the *y* dimension, as shown along the left hand column of Figure 2.5. The combination patterns shown in the rest of Figure 2.5 are obtained by tuning in one harmonic in the x dimension and a different harmonic in y. As in the case of Lissajous figures, these combinations will produce standing waves only

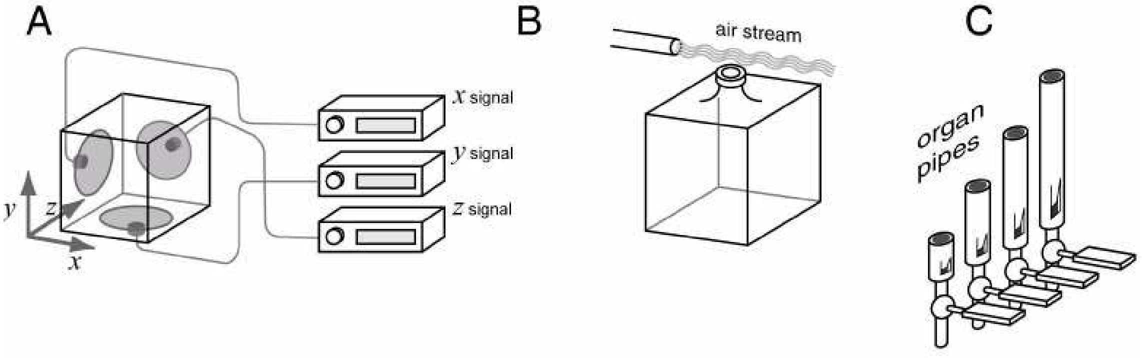

Fig. 2.6. A: A cubical air-filled box, or resonant cavity, with loudspeakers attached to three orthogonal sides, each driven by a signal generator that generates sinusoidal waves at various frequencies. B: A cubical "bottle" is made to sound by blowing air across its mouth. C: A rank of organ pipes tuned to the natural harmonics of the cubical bottle can stimulate the corresponding standing waves in the cubical resonator.

when the frequencies of the x and y dimensions are related by a rational fraction. All other ratios produce a periodic or cyclic pattern of travelling waves instead of standing waves. Even more patterns can be obtained in combination with harmonics in the z dimension, not shown in Figure 2.5.

The same standing wave patterns could also be produced in the cubical cavity much more simply by the same principle by which a note is produced in an empty bottle by blowing a stream of air across its mouth. That is, the box could be equipped with a “mouth” as suggested in Figure 2.6 B, and a stream of air can be blown across the mouth of the cubical “bottle” producing the same repetoire of standing waves as those shown in Figure 2.5. In this case the energy for the resonance is provided by the stream of air blowing across the mouth of the bottle. Acoustical resonances can also be artificially amplified, by recording the sound waves with a microphone, amplifying the signal through an amplifier, and sending the amplified sound out to a speaker near the pick-up microphone. Amplified feedback of this sort is responsible for the harsh screeches or squeals heard when the gain of a public address system is turned up too high. But the microphone and speaker can also be carefully configured to resonate across or within a resonant cavity, amplifying the standing waves that emerge in the cavity. This principle is demonstrated when an electric guitar plays a note that is endlessly sustained due to feedback, that is, the string is vibrating to the sound of its own amplified vibration, a feedback loop that goes from the physical vibration of the string, to an electrical signal which is amplified, back to an auditory sound wave that feeds back to the vibration of the string again. The amplified resonance can now be modulated by changing the resonant properties of the string, that is, by pressing it at different frets to produce different notes. This principle can be demonstrated in a resonant acoustical cavity, like a large bottle or carboy, by placing a microphone and loudspeaker at the mouth of the bottle, sending the amplified signal picked up by the microphone back out to the loudspeaker. By setting the gain just the right, the bottle can be made to howl or wail at one of its many modes of vibration. Small variations in the placement of the microphone and/or speaker, and adjustments of the gain, can make the sound “yodel” between different frequencies, like a bugle playing different notes in the same length of tube, corresponding to different standing wave modes in the resonating cavity.

Synchronization of Distributed Oscillators

The amplification in a harmonic resonance system need not involve only a single amplifier, but can also be performed in a distributed computational architecture using hundreds or thousands of individual local amplifiers distributed uniformly throughout the resonating cavity, that instantaneously amplify and play back the acoustical signal picked up in that local region. This would also tend to amplify the natural standing wave resonances of that resonator while allowing multiple resonances to occur simultaneously. The spontaneous emergence of electrochemical resonance in bulk neural tissue could be explained by this kind of emergent process whereby each cell of the tissue behaves as a local resonator that is capable of resonating at a range of different frequencies, but the frequency at which it resonates at any particular time is influenced by the resonance it picks up from its neighbors. In other words, each cell in the resonating system is a self-amplifying resonator, with a natural tendency to lock into phase with the resonance in the tissue around it. In fact, the tissue of the cardiac muscle has been shown to exhibit exactly this kind of spontaneous resonance. When individual cells of the cardiac muscle are separated from the bulk muscle and maintained in isolation in vitro, they are observed to oscillate electrically, each at its own natural frequency, but when they are assembled into bulk tissue, even if only by contact, they automatically adapt to each other’s oscillations, to produce a single synchronized oscillation of the bulk muscle as a whole. As early as 1953 Bremer (1953) observed spontaneous electrical oscillations of the cat spinal cord that maintain synchronization from one end of the cord to the other, even when the cord is severed and reconnected by contact alone, whereas when completely separated, each fragment oscillates independently. A similar phenomenon is observed when a live snake is chopped into segments, each segment continues to writhe periodically for some time before it eventually dies, whereas in the whole snake the different parts all writhe in unison with a global wave pattern. A similar kind of synchrony between independent oscillators is also observed in the flashing lights of fireflies. Each individual firefly flashes at their own particular frequency when kept in isolation, whereas when released in the open, huge swarms of fireflies all flash in synchrony sometimes with tens of thousands of other individuals. And a similar effect is observed in the chirping of crickets, that also synchronize their chirps with each other. I propose that the principle that synchronizes fireflies and crickets to flash or chirp in synchrony, is the same principle that synchronizes the various parts of a firefly’s brain so as to behave as an integrated whole, a unitary Gestalt, as also observed in visuospatial experience, and in the dynamic symmetry and balance of motor patterns across space and time. This is the emergent self-organizing pattern formation principle exploited by nature for its templates for geometrical forms. The resonance automatically discovers or "computes" the array of harmonics of the resonating system because the resonances feed back on, or amplify themselves, and thus they occur more readily, at a lower energy, than non-harmonic vibrations that tend to cancel each other by destructive interference.

Falstad's box modes applet, and the various examples of resonance described above, represent acoustical standing waves composed of patterns of higher and lower instantaneous air pressure. But the same patterns of standing wave resonance will appear in any volumetric resonating system that has the most general properties required of a harmonic resonant continuum, from acoustical vibrations in a hollow box, to mechanical vibrations in a solid body, to electromagnetic oscillations in a semiconductor crystal, to standing waves of laser light in a lasing optical cavity, or microwaves in a maser cavity, and of course, electrochemical oscillations in neural tissue that project spatial patterns across the brain and nervous system. All of these diverse systems can be contrived to produce volumetric spatial patterns as in Figure 2.5, and each pattern is associated with a specific temporal frequency of oscillation, a musical pitch, or tone, in the case of acoustical vibrations. The frequencies of the harmonic modes of an acoustical box are therefore related to each other harmonically, like the notes in a musical scale. It is the relation between the frequency of a vibration, and the spatial pattern of its standing wave, that allows a frequency encoding of spatial patterns in the brain.

Temporal Frequency Encoding

Falstad's Box Modes Applet displays a row of grids of little squares under the box modes simulation, representing the various harmonics of the box, organized in [*x,y*] grids, and the *z* dimension is represented by different layers of [*x,y*] grids from left to right. The first [*x,y*] grid on the left represents the *x* and *y* modes with *z* = 0; the second grid shows the [*x,y*] modes with *z* = 1, the third grid represents *z* = 2, and so forth to higher harmonics of *z* in successive planes of [*x,y*] grids, representing a cubical [*x,y,z*] array. These squares can be clicked in Falstad's simulation to make the corresponding standing wave patterns appear in the simulation, as shown in Figure 2.7. For example clicking the square [1,0,0] turns on the first harmonic standing wave in *x*, as shown in Figure 2.7 A, while clicking [0,1,0] turns on the first harmonic in *y*, as shown in Figure 2.7 B. Clicking the

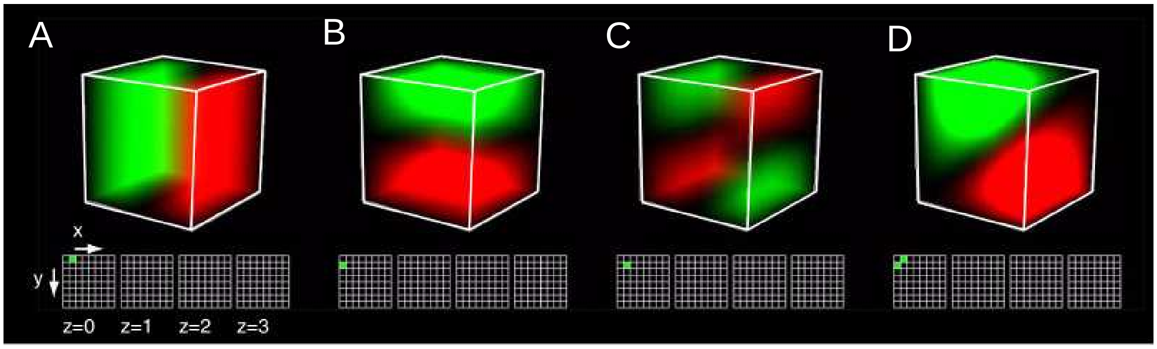

Figure 2.7. Harmonic frequency representation of the standing waves in the Box Modes applet, showing A: First harmonic in x, or [x,y,z] of [1, 0, 0]. B: First harmonic in y, or [0, 1, 0]. C: Cross product of first harmonics in x and y, or [1,1,0]. D: Simultaneous first harmonics in x and y, or [1,0,0] and [0,1,0].

square [1,1,0] turns on the cross-product of those two harmonics, as shown in Figure 2.7 C. The harmonics can also be combined, by turning on [1,0,0] and [0,1,0] both at the same time, as shown in Figure 2.7 D. Note how the combined harmonics in Figure 2.7 D represents an *additive* type of function, or summation of the component patterns; that is, the compound pattern is positive in the quadrant where the sum of *x* and *y* components is positive, and negative where the sum is negative. The cross product in Figure 2.7 C, on the other hand, represents a *multiplicative* or *conjunctive* combination of the components, that is, the polarity reversal in each dimension changes the polarity of the pattern in the other dimension also, producing two polarity reversals, one across *x*, and the other across *y*, with zero-valued nodes located wherever either the *x* or the *y* pattern is zero. The standing waves in the resonant cavity therefore express a unique kind of *spatial logic*, in which the phase of vibration at any point in the volume is determined by a logical AND or logical OR type function on the component harmonics, expressed across extended spatial fields.

There is an interesting *dimensional reduction* in this representation of spatial waveforms by temporal frequencies, which serves as the basis for the principle of symbolic abstraction in biological computation. Imagine that the grids of little squares in Falstad's Box Modes applet are like electronic keyboards, that each produce a single musical note for each key (little square) when pressed (clicked) individually, or harmonious chords when pressed in combination. The keys on these keyboards define a musical scale. For example if key [1,0,0], the fundamental, is the note C, then key [2,0,0] would be higher by an interval of a fifth, or G, 2/3 of the fundamental frequency; key [3,0,0], double the fundamental frequency, would be C an octave above the fundamental; key [4,0,0] would be a frequency of 5/4 of the fundamental, or E, and so forth up the scale. Each note in this harmonious progression corresponds directly to one of the waveforms in the array of standing waves, while harmonious chords produced by playing multiple notes simultaneously correspond to the compound patterns composed of combinations of those harmonic components.

Analog electronic keyboards, like their acoustical counterparts in pianos, harpsicords, and organs, produce their musical notes by a simple one-dimensional resonance in an appropriate resonator: a tuned circuit, a stretched string, or a tuned pipe. A resonance can be established in a resonator either “top-down”, for example by striking or plucking the string with a hammer or pick, or it can be produced “bottom-up”, by simply playing the note to which the resonator is tuned, and thus stimulating a sympathetic resonance in the resonator. For example, imagine a rank of organ pipes adjacent to the acoustical box, as depicted in Figure 2.6 C, whose pipes are tuned specifically to the harmonics of the acoustical box. If a particular standing wave is vibrating in the acoustical box, the vibration will stimulate a sympathetic vibration in the organ pipe whose frequency matches that of the resonance, (especially if amplification is provided at some point in the feedback loop) thus the presence of that particular standing wave pattern in the box is registered or “detected” by the resonance that emerges in the corresponding organ pipe, in the same way that a matching pattern in the receptive field of a neuron in a neural network model stimulates activation in its cell body. The simple one-dimensional resonances in the organ pipes are a *dimensionally reduced*, or *abstracted* symbolic representation of the corresponding volumetric spatial pattern that appears in the box, like a warning light on an instrument panel that registers some vital state of the system to which it is connected, in highly reduced symbolic form. In the case of the resonance model, the relationship between the spatial pattern in the box and its reduced symbolic representation in the organ pipe, is established not just by definition, or by connection, but by the fact that the pattern in the box evokes activation or resonance in the corresponding tube as a “bottom up” recognition of that pattern, and also by that fact that “top down” activation of the organ pipe by blowing air through it, automatically generates a reified exemplar of the corresponding waveform in the acoustical box in Figure 2.6. The extended spatial pattern and its reduced symbolic abstraction are thus intimately coupled in a bi-directional causal relationship in which the presence of either one immediately stimulates a manifestation of the other, even though they are expressed in complementary representational codes. This goes significantly beyond the computational paradigm of the neural network concept of neural activation triggered by patterns in its receptive field, because the bottom-up mechanism behind the abstract recognition automatically includes also a top-down reification of the symbolic abstraction, as an essential aspect of the recognition mechanism. There is no need to define additional top-down neurons with appropriately patterned projective fields, as is often proposed in feedback neural network models, because the feedback through reciprocal interactions is already part of the principle of recognition. And it is not just a simple feedback connection, but an actual forward-and-inverse transformation that expands or reifies the dimensionally reduced symbolic code back into the explicit spatial pattern that it represents by an emergent Gestalt-like process that is characteristic of harmonic resonance.

In the next chapter we will explore the properties of harmonic resonance that make it so useful as a computational and representational principle in biological computation.

Chapter 3

Representational and Computational Properties of Resonance

Information Encoded in a Standing Wave

We have seen that harmonic resonance is a unique pattern formation principle that can create a great variety of spatial patterns based on the twin principles of symmetry and periodicity, and those patterns are automatically sorted, or organized in an ordered hierarchical array, in which the simpler, more primal patterns correspond to the lower vibrational energies, while higher harmonics on those primal basis functions require progressively higher vibrational energies to sustain them. This is the ultimate origin of the Gestalt principle of *prägnanz*, the princple by which the simplest interpretation is the one most likely to be perceived, the perceptual counterpart to Occam's razor. In this chapter we explore those aspects of harmonic resonance that are conducive to exploitation as a computational and representational mechanism in the brain.

There are four distinct types of pattern information encoded in a standing wave. Falstad's Box Modes applet depicts the standing wave as a spatial field of varying magnitude and periodically reversing polarity, a sinusoidal type profile in three dimensions, as shown in Figure 3.1 A, for the same four standing wave patterns as in Figure 2.7. This corresponds to the function $u(x,y,z)$ in the wave equation and in Falstad's Box Modes applet. In a Chladni plate, this corresponds to the instantaneous shape of the steel plate during vibration, or the degree of deviation from its central equilibrium configuration. This is the kind of signal that could be used to define continuous gradient type patterns, for example the continuous shading observed in the coloration of a Siamese cat, with its lighter belly blending smoothly with a darker back and extremities. In motor control, the analog magnitude signal might define the dynamic motor field, or pattern of contraction and extension of the continuous slab of muscle of a swimming fish or eel, with analog waves of muscular contraction travelling continuously from head toward tail, or in static form, the posture of an elephant trunk, swan neck, or snake body, that is held frozen in a sinusoidal or curved stance.

A variation on the analog wave function is the analog *magnitude* of the wave function, $|u(x,y,z)|$, shown in Figure 3.1 B, preserving its amplitude but discarding the polarity information. An approximation to this pattern can be observed in Falstad's Box Modes applet by setting the simulation speed so fast that the pattern flickers rapidly between green and red regions, creating the impression of a three-dimensional pattern of yellow (red + green) lobes separated by dark nodes. This converts the red/green (alternating with green/red) polarized pattern of the first harmonic waveform, for example, to a twin-hump yellow/yellow pattern across the zero-amplitude node. This spatial pattern is also encoded in the standing wave signal where it is readily available for detection through a simple absolute value function.

The standing wave also encodes a stark binary pattern in the sign, or polarity of the standing wave pattern, that is, $sign(u(x,y,z))$, that evaluates to +1 where $u(x,y,z)$ is positive, and -1 where it is negative, preserving the phase of the spatial pattern but ignoring its magnitude. This function can be approximated in Falstad's simulation by turning up the brightness control to maximum brightness, as shown in Figure 2.8 C.This is the kind of signal that would be used to define patterns like the sharply delineated black and white stripes of a zebra, and in motor control, this corresponds to the abrupt all-or-nothing type of motion seen in the jerky robotic bobbing motions of a walking bird, or the jerky movements of our eyeballs as they dart rapidly between fixations with abrupt starts and stops. This aspect of the spatial pattern is also readily readable from the standing wave with the use of a simple polarity or threshold function.

Finally, the standing wave also encodes a fourth pattern, that is the pattern of *nodes* of the standing wave, that is, the lines, surfaces, or vertices across which the polarity of the wave reverses, as shown by the gray planes in Figure 2.8 D. The nodal pattern is like an outline drawing that uses thin lines to depict the abrupt transitions between uniform patches of color in a cartoon. The pattern of nodal surfaces can also be seen as the *eigenfunctions* of the pattern, the centers of symmetry about which the pattern of positive and negative volumes alternates, the only regions that do not themselves alternate in polarity. The significance of this nodal representation of pattern will become clear in later chapters. All four of these patterns are encoded implicitly by the standing wave, where they can be read out locally at any point in the volume by simple local detectors that can be replicated throughout the volume of the resonating system, like the units that translate the chemical standing wave in morphogenesis to patterns of colors in animal skin, or to volumetric patterns of specific tissue types in the developing embryo, and like the cells of the cardiac muscle, or the cilia of a paramecium, that transduce patterns of electrochemical oscillations into mechanical contractions.

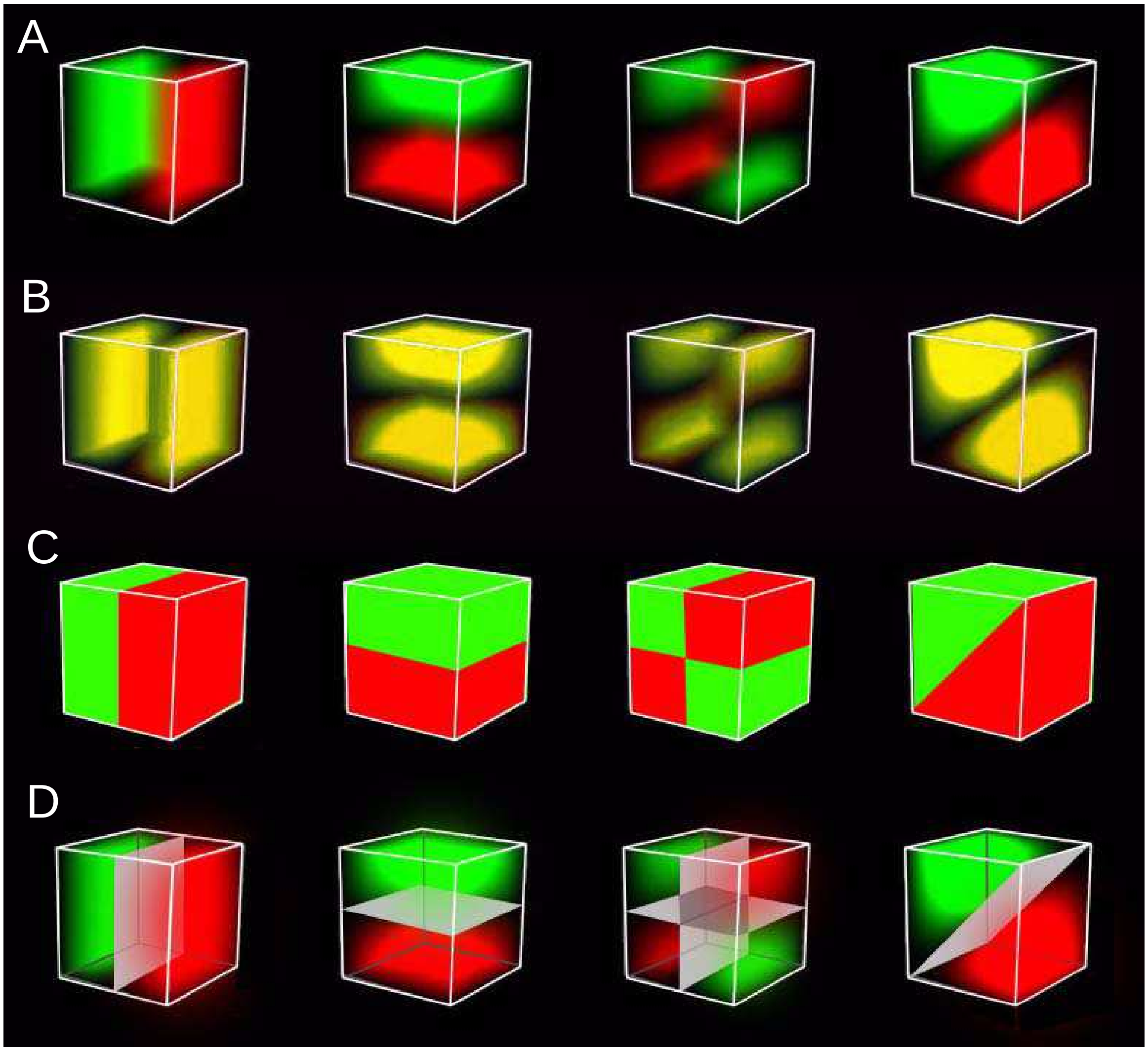

Figure 3.1. Four kinds of spatial information are present in the standing wave, which can be read out by simple local sensors distributed throughout the volume of the representation. A: An analog magnitude and phase. B: The absolute value of the magnitude, discarding phase. C: A binary polarity signal that discards magnitude but preserves phase. D: Nodal planes within which the vibration of the standing wave is zero.

Sync Pulse to Read Alternating Signal

The standing wave pattern is actually in continuous oscillation, reversing polarity with every cycle. A simple local detector somewhere in the volume of the resonator would thus record an oscillating signal rather than a static pattern. In order to read out the static pattern represented by the standing wave, the detector must operate in phase with the oscillating signal so as to read it only in one phase of the cycle and not the other. This principle is demonstrated for example in the photographs of Murray's vibrating steel plates, shown in Figure 1.8. The patterns of light and dark shades in that figure were obtained by constructive and destructive interference between the light illuminating the vibrating steel plate, and that reflected back from the plate, which in turn depends on the instantaneous pattern of deflection of the plate as it vibrates. Simple viewing with the naked eye or a simple camera would reveal no pattern at all, due to the rapid reversals of the light and dark regions of the pattern many times each second. In order to capture only one phase of the pattern and not the other, as seen in the figure, it is necessary to either snap the picture in a fraction of a second at the peak of the oscillation, or, to open and close the shutter in synch with the oscillation of the pattern in order to integrate light during one half-phase only. More generally, the standing wave pattern can be read out from a harmonic resonance representation by local detectors that detect periodically in phase with the oscillation. In other words, each simple detector distributed throughout the volume of the resonance representation must have access to two signals: the local oscillating standing wave signal itself whose waveform represents the signal, and a universal synchronization signal, or strobe pulse, that acts as a "read enable" signal, to turn on all the detectors in the volume only when the strobe pulse is in positive phase. The output of this conjunction, or logical AND between signal and strobe, defines an enduring static signal of volumes of positive and negative magnitude, distributed throughout the volume of the resonator as a wave function. While the standing wave pattern itself varies across the volume of the representation, the strobe signal is global, turning all the sensors in the volume of the representation on and off simultaneously in synchrony.

To take a concrete example in order to sharpen our mental image of this principle, imagine an array of tiny microphones suspended on an array of fine wires that form a 3-D lattice throughout the volume of the acoustical box described above, as suggested in Figure 3.2 A, in a way that does not interfere too much with the acoustics of the box. Each microphone records the local oscillations of air pressure at that point in the volume, and transduces it to an alternating electrical voltage that drives a simple circuit, as depicted in Figure 3.2 B, creating a current that flows clockwise around the circuit during the positive phase of the acoustical oscillation, and counter-clockwise during the negative phase. Two side loops connected to this circuit are equipped with light-emitting diodes (LEDs). Besides emitting light, the LED is also a *diode*, that is, a one-way valve that allows current to flow in one direction but not the other, like a check valve in a hydraulic or pneumatic system, or a ratchet in a mechanical system. In this circuit the two LEDs are biased in opposite directions, so that one side-loop allows current to flow only during the clockwise phase of the current, the other only during the counter-clockwise phase. Let us say that the clockwise current LED is green, and the counter-clockwise one is red. An acoustical standing wave vibration of say 30

Hz, picked up by the microphone, would thus cause the green LED to flicker at 30 Hz in synch with the positive phase of the wave, while the red LED would flicker at 30 Hz in counterphase to the green one during the negative phase. The rapid alternation of red and green would create an impression of yellow (red + green in additive color mixing) as shown in Figure 3.1 B. If the volume of the box were filled uniformly with hundreds of tiny LED circuits of this sort, then an acoustical standing wave in the box would manifest itself by a glowing field of LEDs everywhere throughout the box as a function of vibrational amplitude, leaving dark regions only at the nodes of the vibration where the acoustical vibration is zero. In other words, this circuit reveals the magnitude function |*u*(*x*,*y*,*z*)| of the standing wave.

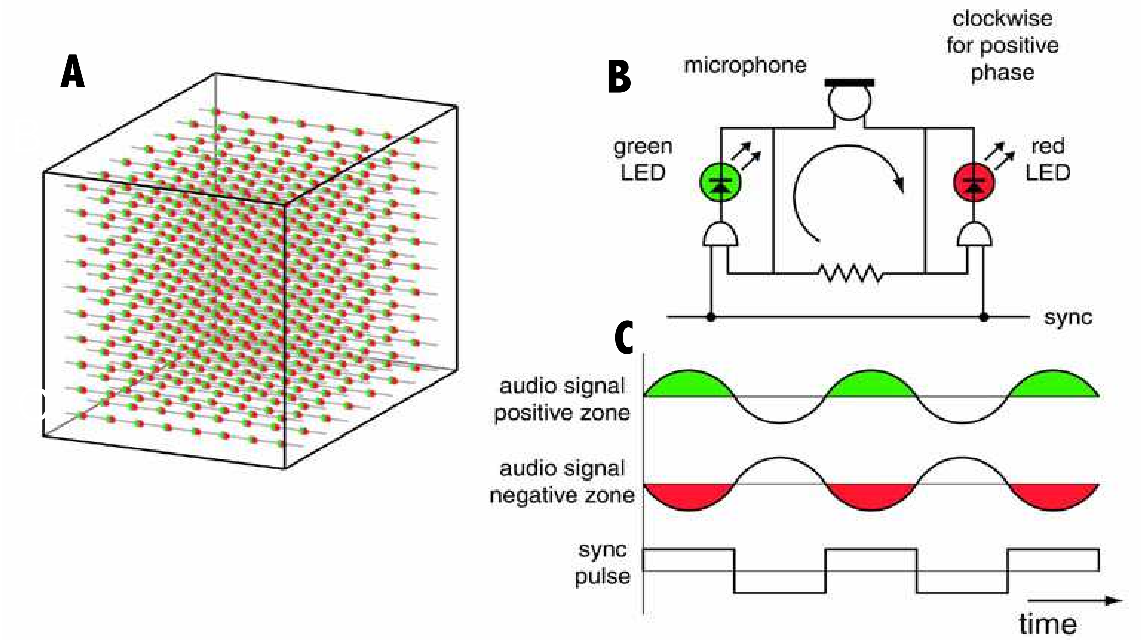

Figure 3.2. A: grid of tiny electrical circuits strung on an array of fine wires throughout the acoustical box, each consisting of B: a tiny microphone and two LEDs, wired so that the green LED lights up in the positive phase (clockwise current), the red one lights up in the negative phase. C: A sync pulse or strobe signal oscillating in phase with the standing wave serves as a “read enable” line to integrate only during the positive phase, turning on the green LEDs in positive zones, and red LEDs in negative zones of the standing wave.

Now to introduce a common strobe signal, let us say that the grid of wires on which the LED circuits are suspended is connected to a sync pulse that feeds all the circuits simultaneously with the same alternating sync signal, and this sync pulse is connected to the side-loop circuits by AND gates, that allow the side-loop current to flow only during the positive phase of the sync pulse, but blocks the flow during the negative phase, as shown in the circuit diagram in Figure 3.2 B. If the sync pulse is now alternated at the same 30 Hz as the acoustical standing wave, as suggested in Figure 3.2 C, it will turn on the green LED circuits during the positive phase of the oscillation in the positive zones of the standing wave pattern, but shut off the red LED circuits during the negative half-cycle, resulting in a static or flickering pattern of green lights only in the green zones of Figure 3.1 A, whereas in the negative zones of the standing wave the sync pulse will turn on only the red LEDs during the positive phase of the sync pulse, and thus this simple local circuit replicated throughout the volume of the resonator will serve to light up the wave function $u(x,y,z)$ as volumes of glowing green and red, as shown in Falstad's box modes applet. If the sync pulse is phase-shifted to be positive during the *negative* phase of the standing wave, this will reverse the patterns of red and green volumes to paint the negative lobes green and the positive lobes red, and shifting the phase to intermediate values will reveal a phase-shifted pattern of red and green zones.

The sync pulse can also be used to pick out or tune for particular components of an oscillating signal. For example given a compound standing wave composed of a fundamental oscillation along with one or more higher harmonics, when probed with a sync pulse at the fundamental frequency, would reveal only the fundamental component of the oscillation, not its higher harmonics, whereas when tuned to the frequency of one of the higher harmonics it would pick out the waveform of that harmonic component alone. The sync pulse can thus be tuned somewhat like a radio receiver to pick out the component or components of interest. This principle of tuning for features by oscillation frequency is also observed in the interactions between harmonics components of different frequencies in a compound oscillation.

Static and Dynamic Representation of Space and Time

One of the most powerful features of a harmonic resonance representation is that the representation of spatial patterns by a standing wave automatically and inevitably also includes a representation of spatiotemporal patterns that are cyclical over time, achieved by oscillations out of synch with the fundamental resonance. Let us investigate this dynamic aspect of a harmonic resonance representation using Falstad's Box Modes applet again. The reader is encouraged to run the applet and follow along with the examples described below.

We begin with a first harmonic oscillation in x, by clicking the box [1,0,0] in the applet. This produces an alternating red/green green/red pattern that represents a single static waveform that alternates across a fixed static node plane. Clicking box [2,0,0] produces a second harmonic red/green/red pattern alternating with green/red/green, again, alternating across (this time) two static node planes, and oscillating at a higher frequency. If you turn on both [1,0,0] and [2,0,0] at the same time however, as shown in Figure 3.3 A, then something strange occurs, due to the different temporal frequencies of the two components. The observed pattern alternates between green/red/dark, red/green/red, dark/red/green, as suggested in Figure 3.3 A through C, and then back again in reverse order, as suggested in Figure 3.3 C through A. There is a discrete, or stepwise character to this motion, as the waveform hops abruptly from one mode to the next.

To understand the meaning of this pattern of oscillation, now turn on all of the higher harmonics of this same series, that is, click on the squares [1,0,0] through [7,0,0], as shown in Figure 3.3 D. These nodes represent frequencies that are 2, 4, 8, 16, 32, and 64 times that of the fundamental, or a series of octaves of the fundamental. The dynamic pattern now begins with a plane of positive green within the y/z plane, that sweeps progressively from left to right along the x dimension, as in Figure 3.3 D, E, and F, and when it reaches the end, it reverses direction and proceeds back from right to left again, as suggested in Figure 3.3 F, E, and D. This is a highly structured and orderly pattern of behavior across space and time, a reciprocal sweeping back and forth of a positive plane through an otherwise negative volume, and the sweep occurs in a series of eight discrete steps or jumps, rather than a continuous motion, and those discrete jumps can also be seen as ghostly “echos” of the plane at discrete intervals across the negative zones in Figure 3.3 D, E, and F. If you click off the higher harmonics of this resonance, i.e. shut off first [7,0,0] then [6,0,0] etc., you will see a progressive reduction in the number of steps in the motion, and at the same time a reduction in spatial resolution, that is, the plane of positive value becomes progressively thicker, so as to span the same volume in fewer steps. Conversely, adding higher harmonics increases the spatial and temporal resolution, producing (in the limit) a perfectly continuous sweep of an infinitely thin plane. What we have here is an explicit, extended spatiotemporal pattern that is reified across space and time, represented by a static combination of harmonic frequencies, and the spatial and temporal resolution are simultanously altered by the addition or removal of higher harmonics. This is exactly the kind of pattern that would serve as a spatiotemporal pattern primitive, the dynamic equivalent of the static waveform functions in Figure 2.5 that serve as a basis set for static patterns. For example this kind of resonance could serve top-down as a motor control signal to produce coherent waves of contraction in a slab of skeletal muscle, or peristaltic contractions along a length of digestive tract, or synchronized waves of moving cilia on the surface of a simple creature, and the same principle could serve bottom-up as a sensory function to detect waves of sensory stimuli in those same patterns, that is, an oscillating bar of light in a visual stimulus, or oscillating tactile sensation in a somatosensory stimulus, where the dynamic sensory stimulus lights up a static pattern of nodes whose phase differences represent that dynamic pattern.

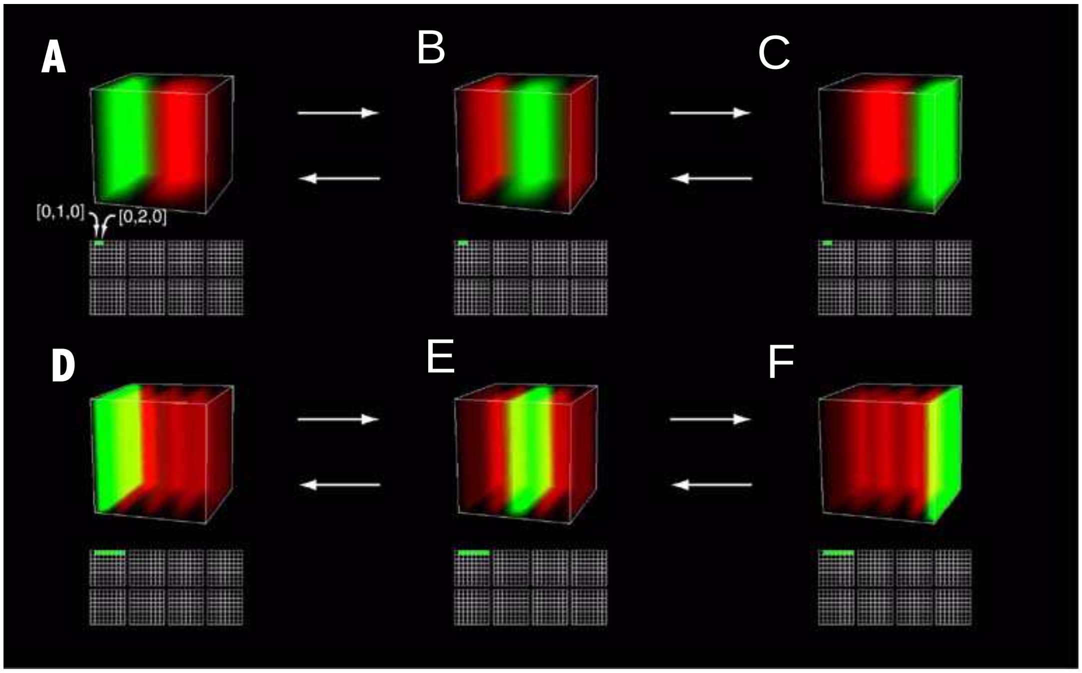

Figure 3.3. A: Turning on the first and second harmonics in *x* produces B: a green slab on the left, with red elsewhere, that moves in three discrete steps to the right, then back again to the left. C: Turning on the higher harmonics of this same pattern produces D: a thin slab of green, with red elsewhere, that also sweeps alternately left and but this time in eight discrete steps, a remarkably structured or ordered pattern of regular reciprocating motion.

Now let us try a different combination of harmonics, turning on the boxes [1,0,0] and [3,0,0] simultaneously, as suggested in Figure 3.4 A through C. This produces an interesting variation on the dynamic pattern above. This time a pair of red and green planes appear on opposite sides of a central node plane, and each plane oscillates back and forth across one half of the cube in mirror symmetry with the other, like two hands clapping, both reversing polarity as they meet at the center. In other words, the pattern goes through the sequence of Figure 3.4 A, B, and C, then it continues through two more stages, (not shown) that we could call D and E, which are mirror-reversed replicas of patterns C and B; then the sequence reverses again through E, D, C, B, and back to A again, and the whole forward and reverse cycle repeats indefinitely. Again, addition of higher harmonics in the same pattern, as shown in Figure 3.4 F through H, (and two more stages I and J, not shown) increases the resolution of this dynamic pattern across both space and time, while preserving the same basic pattern. With the addition of the

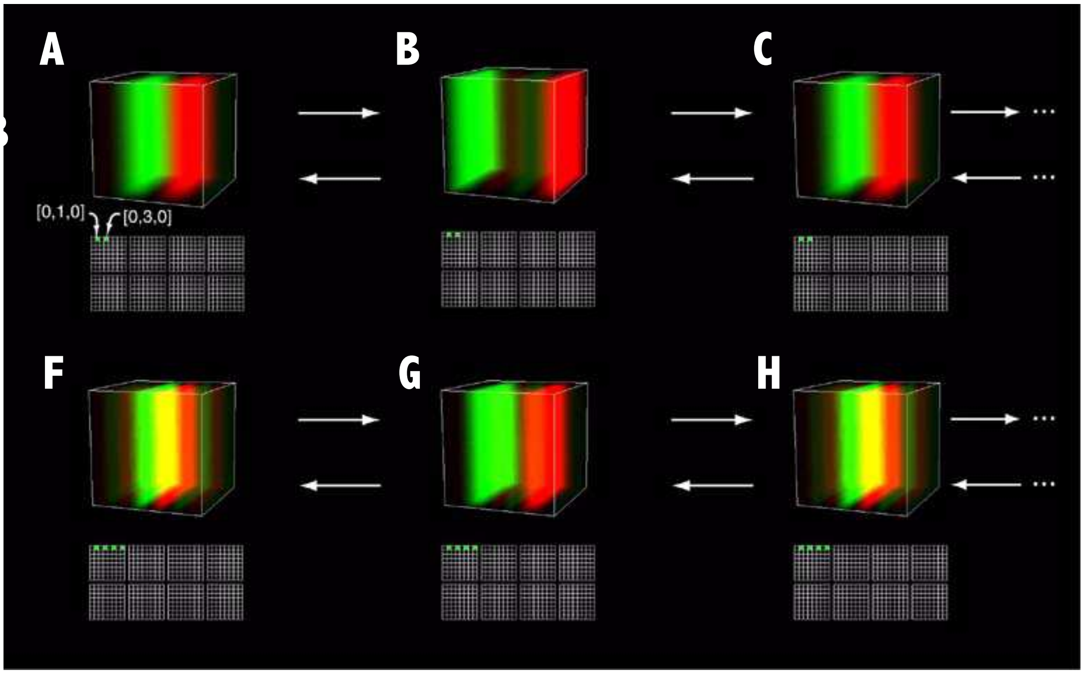

Figure 3.4. Turning on the first and third harmonics in *x* produces A: a pattern of green and red planes at the center, that jump B: left and right out to the ends, and C: back to the center, where they reverse color, and jump to the ends, then back to the center again (not shown) in endlessly repeating cycles. C: Adding also the higher odd harmonics (fifth and seventh) produces D through F: a refined version of the same inward and outward pattern, this time sweeping almost continuously in four little jumps out to the ends and back again.

higher harmonics, the pattern increases in resolution both in space and time, that is, the positive and negative planes get thinner, and the transition between F and G occurs in four discrete steps, instead of just two, as in A and B.

These same patterns of simple motion can of course be replicated in the y and z dimensions by the same principle, as the reader can easily verify with Falstad's simulation. Even more interesting is to observe the combinatorial patterns that emerge with simultaneous patterns in two dimensions. For example turning on the harmonics [1,0,0] through [7,0,0] at the same time as [0,1,0] through [0,7,0] creates two planes at right angles to each other, sweeping back and forth across the x and y dimensions simultaneously, and adding nodes [0,0,1] through [0,0,7] creates three sweeps at right angles to each other, an extraordinarily orderly and systematic pattern to emerge from such a simple resonating system. (Note: Falstad's simulation is arbitrarily limited to 10 harmonics presented simultaneously, so only lower resolution approximations to these patterns can be actually achieved in the simulation, although this limitation does not apply to a real resonance mechanism).

An even more intriguing pattern is obtained by the cross-product combination of harmonics across *x* and *y*, that is, by turning on nodes [1,1,0], [2,2,0], [3,3,0] ... up to [7,7,0], as shown in Figure 3.5. This produces a peculiar compound sweep pattern that begins with a green plane of positive values in the diagonal *x* = *y* plane, as in Figure 3.5 A, that splits into two parallel planes connected on two sides like an open-ended box, as in Figure 3.5 B, that expands outward in opposite directions as a hollow box whose short sides grow as its long sides shrink, as in Figure 3.5 B and C, ending in a diagonal plane in the opposite direction (in the *y* = 1-*x* plane, shown in Figure 3.5 D) and then the pattern turns around and repeats in reverse sequence from D back to A, and the alternately reversing sequence continues in cyclic oscillation. Another extraordinarily complex and lawful emergent pattern from very simple parallel resonance principles, a two-dimensional generalization of the one-dimensional reciprocating sweep of the last example.

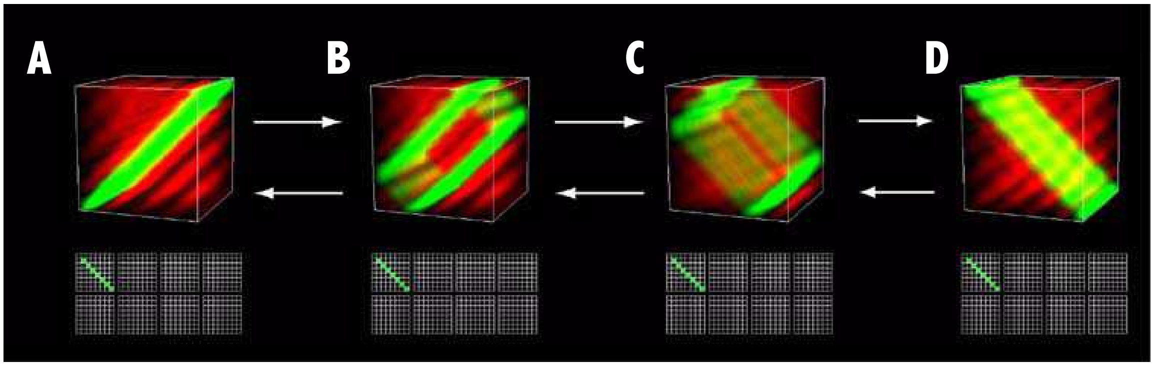

Figure 3.5. A: Turning on the [x,y] cross-product harmonics produces B: a diagonal green plane that expands outward like a hollow box then collapses back to a diagonal in the other direction, before reversing the sequence and returning to the initial configuration, and then repeating endlessly.

Finally, a still more interesting pattern is obtained by the full cross-product terms across *x*, *y*, and *z*, that is, by turning on nodes [1,1,1], [2,2,2], [3,3,3] ... up to [7,7,7], as shown in Figure Figure 3.6 A through D. This produces a strikingly intricate pattern of orthogonal sweeps back and forth between oppositely-oriented tetrahedra that span the opposite corners of the cube. The bright triangle in Figure 3.6 A is the near face of a tetrahedron, with corners at points {0,0,0}, {1,0,1}, {0,1,1} expressed in unit cube coordinates of the spatial domain, with the opposite vertex of the tetrahedron located at {0,0,1}, seen transparently through the triangular front face. (Note: {*curly braces*} are used to indicate coordinates in the *spatial* domain of the standing wave resonance in the box, while [*square brackets*] indicate harmonics in the *frequency* domain in the grid of little squares.) Each of the four triangular surfaces of the tetrahedron exhibits a bright green (positive)

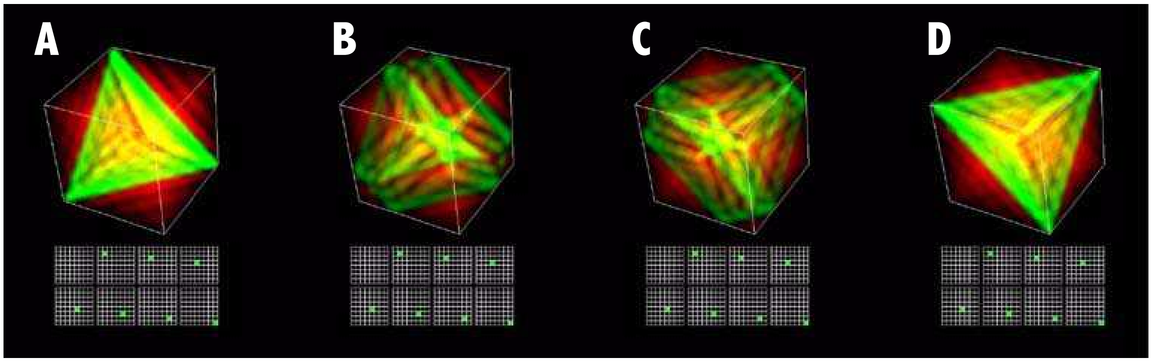

Figure 3.6. Turning on the [x,y,z] cross-product harmonics produces at first A: a tetrahedron. All four faces sweep toward their opposite vertices simultaneously, (B through D) crossing and reversing in the middle, producing a tetrahedron the other way, (D), with faces and vertices reversed, after which the pattern evolves back again in reverse sequence from D through C, B, back to A again, and so on in endless succession.

value throughout its surface, with red (negative) elsewhere, both inside and outside the tetrahedron. The four surfaces sweep simultaneously in a direction normal to their planar surface toward the opposite vertex, as shown in Figure 3.6 B through D, in a three-dimensional generalization of the growing/shrinking box of Figure 3.5, with an additional third dimension of symmetry in space and time. The final shape in Figure 3.6 D is inverted relative to the initial tetrahedron, with vertices now spanning the cardinal corners of the unit cube, that is, {1,0,0},{0,1,0},{1,1,1} and {0,0,1}.

The nature of this pattern can be analyzed by turing on its first component all by itself, that is, by clicking the node [1,1,1] alone. This marks the *cardinal* corners of the *spatial* cube in the resonance representation with positive polarity, that is, corners {1,0,0}, {0,1,0}, {0,0,1} appear green, while marking the opposite, complementary, or alternate corners as negative, that is, corners {0,1,1}, {1,0,1}, {1,1,0} appear red. If you allow the simulation to run, you will see these opposite corners alternate in polarity, the positive corners turning negative, and vice-versa;. Now turn on the next harmonic in the frequency representation, that is, click box [2,2,2] along with [1,1,1]. This serves to connect the positive corners with each other to form a fuzzy tetrahedron with unit-vector corners {1,0,0}, {0,1,0}, {0,0,1}, and negative elsewhere. Clicking still higher order harmonics [3,3,3], [4,4,4], etc. preserves the same tetrahedral shape, but refines its spatial resolution to a crisp geometrical form. Now if you allow the simulation to run, you will see each of the four sides of the tetrahedron sweeping as a plane travelling in the direction of its surface normal, shrinking to a point at the opposite vertex of the tetrahedron, from whence it sweeps back again from the apex back to the base, and back again in endless cycles, all four sweeps occuring across all four faces and their opposite corners of the tetrahedron simultaneously, a remarkably beautiful and intricate pattern of mirror-symmetrial motions, whose complexity emerges spontaneously from a simple resonance in a box. In fact, this is just a generalization of the pattern of motion shown in Figure 3.5 into the third dimension.

Combinations of Different Dimensions

Other intricate patterns can be obtained by combinations of harmonics in different dimensions. The second harmonic pattern obtained by clicking [2,0,0] for the *x* dimension, or [0,2,0] for the *y*, creates a red/green/red or green/red/green "sandwich" pattern in the corresponding dimension, as seen for the x dimension in Figure 3.7 A. If you click both [2,0,0] and [0,2,0] simultaneously, as shown in Figure 3.7 B, you get a combination pattern that is a central cylinder surrounded by four corners of opposite polarity, as seen in Figure 3.7 B, a two-dimensional generalization of the center-surround contrast of the corresponding one-dimensional patterns. (Actually, the "cylinder" has a diamond-shaped cross-section rather than a circular one, as can be seen by increasing the brightness to approximate a polarity mapping of the pattern of *sign*(*u*(*x,y,z*))) If you now click [0,0,2] along with [2,0,0] and [0,2,0], as shown in Figure 3.7 C, you get a three-dimensional generalization of the center/surround concept that appears as a central sphere (actually an *octahedron*, or two pyramids attached base to base) surrounded by opposite-polarity corners. These spatial patterns are static, that is, the nodes remain fixed, although the positive and negative regions reverse alternately about those static nodes, because the *x*, *y*, and *z* components oscillate at the same frequency.

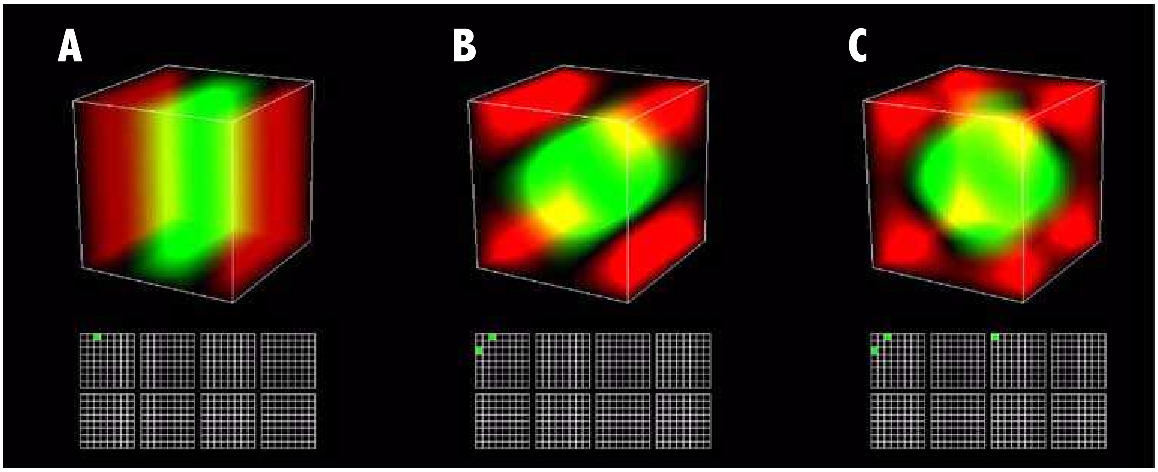

Figure 3.7. A: The second harmonic in *x* produces an on-center off-surroudn sandwich pattern. B: Adding the second harmonic in *y* produces a two-dimensional center/surround as a central green cylinder surrounded by red. C: Adding also the second harmonic in *z* produces a three-dimensional center-surround, a central green sphere encircled by a red surround.

Rotation Through Relative Phase Adjustments

Falstad’s Box Modes applet also allows you to change the relative phases of the component harmonics by dragging the mouse horizontally. This is exactly analogous to shifting the phase of the sync pulse as described above. When you click one of the little boxes in Falstad’s simulation, it toggles the corresponding frequency node on or off with each click, with the phase set to zero, so when you click more than one box, they are automatically in phase with each other. But you can also click and drag the mouse horizontally in the little click box to shift the phase of the pattern from 0 to 2π, which is indicated in the simulation by a tiny blue line that scans across the little click box like a tuner needle on a radio dial. For example if you click box [1,0,0] with the simulation stopped, to create a first harmonic green/red pattern, then drag the phase with the mouse, the waveform in the resonator will shift in phase with your mouse drag, which converts the green/red pattern to a red/green one, with a zero amplitude point in between the two opposite phases. With the simulation running, this has little practical effect on a single harmonic waveform when played by itself, but relative phase has a profound effect when the waveform is composed of two or more components.

(Falstad’s Box Modes applet also allows you to adjust the *magnitude* of the wave by dragging the mouse vertically in the little boxes. This also has the effect of reversing the polarity of the waveform, because a magnitude of -1 swaps the positive and negative regions. We will be leaving the magnitude alone for the following demonstrations, and mention it here only to prevent confusion if this feature is encountered accidentally.)

Let us begin with a pattern of [1,0,0] and [0,1,0] simultaneously, as above, both clicked to phase zero. This produces a red/green pattern across a 45 degree angled nodal plane, as shown in Figure 3.8 A. Turning the individual components off and on again in turn, shows how the horizontal and vertical green/red patterns combine to create the diagonal one. Now click node [0,1,0] and drag it sideways to shift its phase. If the simulation is stopped, this will progressively change the angle of orientation of the node plane separating red and green regions, as shown in Figure 3.8 B, C, and D. If the simulation is running, however, it causes the entire pattern to rotate continuously, either clockwise or counter-clockwise, depending on the exact phase shift. As the phase shift is slowly increased from zero, the pattern rotates first counter-clockwise, and the rate of rotation rises to a peak at phase difference of 90 degrees, when the component waves are in *quadrature*, then reduces back to a stationary pattern as the phase difference passes

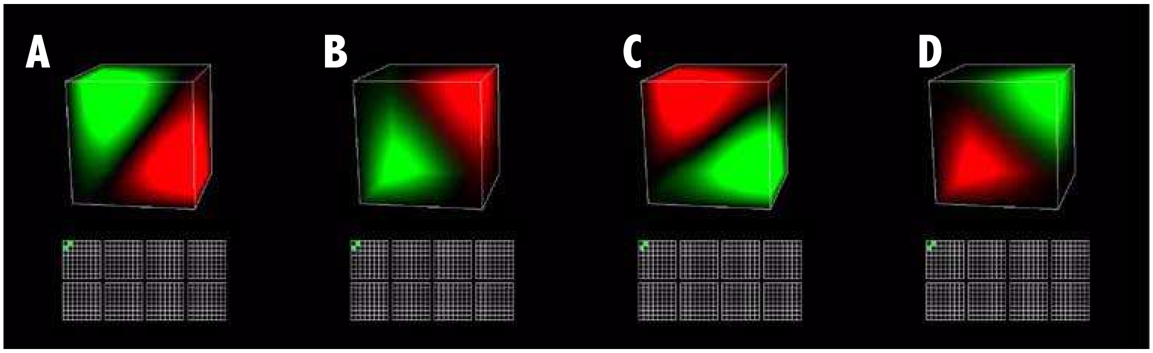

Figure 3.8 A: Turning on the first harmonic in x and y creates a green/red pattern oriented at 45 degrees. B through D: Advancing the phase of the y component sets the pattern into rotation at a rate that is proportional to the phase difference between the two oscillations, that reaches a maximum at quatrature (phase difference of 90 degrees), drops to zero at counterphase (180 degrees), then rotates in the opposite direction to a maximum at negative quadrature (270 degrees) and back to zero when back in phase.

through180 degrees. If the phase is advanced still farther, the rotation begins again this time in the opposite direction, clockwise, as the phase of the *y* component is now retarded rather than advanced relative to the phase of the oscillation in *x*, reaching a maximum rotation rate when the waves are in negative quadrature. And if the phase is shifted still farther the phase shift returns to zero, and the rotation slows then stops. What we have here is a representation of a dynamic rotating pattern by way of a static phase relation between waveform components. Trigger two oscillations with the right frequency and phase relation, and you get a rotating pattern. This kind of dynamic pattern generation principle could be used for example in motor control, to produce a rotary pattern of muscular contraction, and in sensory systems, to capture or characterize the rotation of a dynamically cycling waveform pattern by the phase relation between its static harmonic components. In this example the rotation occurs through the *x,y* dimension. Clicking the z node [0,0,1] tilts the red/green divide 45 degrees in the *x,z* and *y,z* dimensions, and shifting its phase sets the pattern into rotation across the *z* dimension also.

Higher harmonics produce more complex spatial patterns, and phase shifting them produces more complex patterns of motion also. For the next example clear all active nodes, and click nodes [2,0,0] and [0,2,0] again, to produce the two-dimensional center-surround pattern in Figure 3.9 A, which is the same as Figure 3.7 B above. Now with the simulation stopped, progressively shift the phase of the *y* component [0,2,0] and you will see a progression of the pattern as shown in Figure 3.9 B through D. At first, the central green cylinder elongates vertically to an elliptical cross-section, as do the surrounding red zones, which link up vertically, as in Figure 3.9 B, and this pattern continues to morph into a simple red/

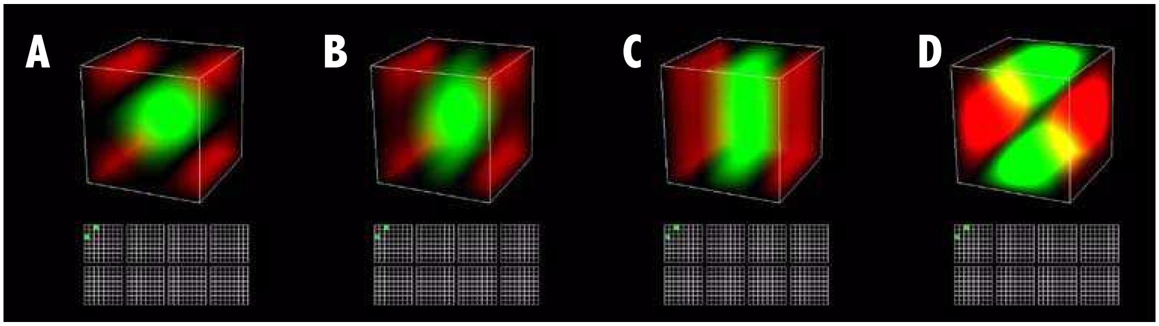

Figure 3.9. Starting with A: a second harmonic on-center off-surround in x and y, the phase of the y component is varied from B: a vertical elongation, to C: a center/surround in the x dimension only. Eventually the pattern shifts to D: positive lobes vertically and negative lobes horizontally. These are all spatial permutations and combinations of the on-center off-surround concept in different dimensions.

green/red sandwich of vertical slabs, as in Figure 3.9 C. This is the point where the phase of the y component is just reversing polarity, and thus the pattern is determined entirely by the x component alone, as can be easily verified by clicking the x node [2,0,0] off momentarily, and seeing pattern disappear altogether. Shifting the phase further still produces the pattern in Figure 3.9 D, with two red lobes left and right, and two green lobes top and bottom. Each of these phase shifts of the static waveforms produce their own characteristic combination pattern. But every combination also has a unique spatiotemporal pattern of cyclic evolution.

With the simulation stopped, re-adjust the phase of the y component, [0,2,0] until you get the central cylinder pattern in Figure 3.9 A. If you allow the simulation to run, the pattern alternates with its inverse across a static pattern of nodal planes. Now shift the phase of *y* a little farther again to get the elliptical cross-section stretched cylinder, and let the simulation run. Now we have a strange circulatory motion where pairs of blobs from left and right merge horizontally into a single blob at the center, which then splits vertically to a pair of blobs one above, one below, that move up and down respectively, away from the center, to be replaced by the next pair of blobs coming in from left and right to merge at the center, and so on round and round, joining horizontal, and splitting vertical in endless cycles. If you observe the behavior of the nodal surfaces that separate red and green volumes, there are four axes of rotation about which four nodal planes rotate in synchrony. And if you shift the phase of y to appear as in Figure 3.9 D, and then let the simulation run, the pattern will once again alternate between opposite contrasts across static nodal surfaces. The points where the nodal pattern is static are the points where the x and y component oscillations are symmetrically balanced in phase, whereas a phase that is advanced or retarded creates rotation either clockwise or counter-clockwise, respectively. The maximum rotation rate that can be achieved in this kind of representation is obtained with waves that are in perfect quadrature, that is, with a phase difference of π/2 or 90 degrees, at which point the rotation rate of the combined x/y pattern is equal to one full circle for every cycle of oscillation of the second harmonic oscillation, which in turn is four times the frequency of the fundamental. With rotational velocity expressed by the amount of phase lag and lead between two waveforms, it is natural for the phase difference to increase to a maximum at quatrature, diminish to a minimum again in 180 degree counterphase, and then build up again to a maximum in the opposite direction at negative quatrature before diminishing to zero again in-phase.

Waveforms that are composed of different frequencies in different dimensions always produce dynamic patterns of motion. For example clicking nodes [1,0,0] and [0,2,0] creates an oscillating pattern that morphs cyclically between the first harmonic in x pattern, green/red (or its complement) horizontally, as in Figure 3.10 A, and the second harmonic y pattern, a vertical red/green/red sandwich pattern shown in Figure 3.10 D, passing through the intermediate stages in Figure 3.10 B and C, first left-to-right (A, B, C, D), then it continues through three more stages (not shown) which we could call E, F, and G, that are the exact mirror images of C, B, and A respectively, ending in a red/green pattern that is a mirror image of the green/red initial pattern, then the whole sequence plays back in the reverse direction again, G, F, E, D, C, B, A, and the cycle repeats alternately forward and backward in sequence indefinitely. This pattern of motion is best understood by observing the nodal planes that separate the red and green volumes, as suggested in the overlay. The nodal plane begins as a vertical surface between

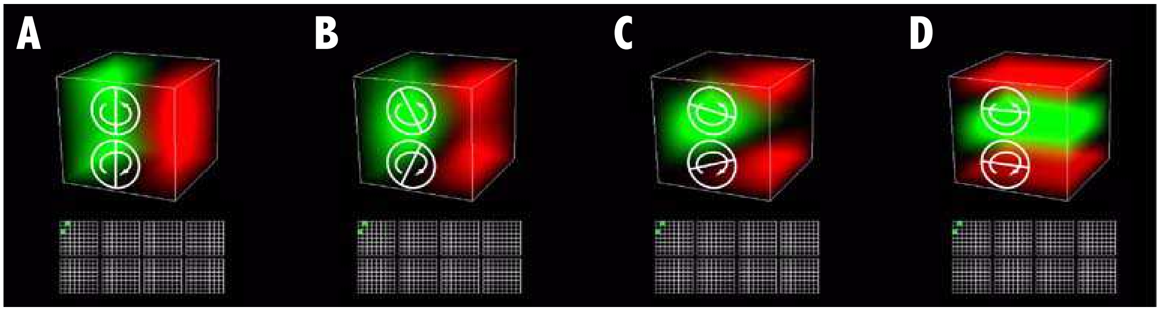

Figure 3.10. A first harmonic in x, along with a second harmonic in y, produces an oscillating pattern that begins as A: two parallel slabs, B: one bulges as the other shrinks back until C: the bulger cleaves the shinker in two pieces that move up and down respectively culminating in D: a second harmonic center/surround sandwich in y. The full cycle would continues four more frames to the right (not shown) in exact mirror symmetric pattern, ending with parallel slabs in the opposite polarity. It then reverses direction and cycles through all the same patterns in the reverse sequence, and then repeats indefinitely.

red and green (A), then folds in the middle about a horizontal axis (B), the upper half rotating counter-clockwise, the lower half clockwise, as indicated by the arrows. As the rotation continues (C), the nodal planes become parallel (D), and the rotation continues through a full 360 degrees above and below, through E, F, and G (not shown), then a full 360 degrees back the other way through G, F, E, D, C, B, and A, totally inverting the volumes of red and green that they separate. In other words, this pattern represents symmetrically opposed rotations around two vortices in reciprocating alternation.

In this experiment there was no component in the z dimension, and thus the pattern is constant or unchanging with differences in depth, a straight projection of the 2-D pattern across *x* and *y* through the *z* dimension. We can describe the progression of patterns in Figure 3.10 A through D as a green and red slab; the green slab folds into a wedge, while the red slab splits into an anti-wedge, filling the space not occupied by the wedge; the angled surfaces of the wedge rotate to a parallel sandwich configuration as shown in D and beyond. We can now modulate the *z* dimension to generalize this pattern to a full three-dimensional shape. If we click on [0,0,1] along with [2,0,0] and [0,2,0], this first harmonic in *z* is of the same frequency as the first harmonic in *x*, and thus oscillates in phase with it, rotating the pattern of reciprocating wedge and anti-wedge to run from one corner of the cube to the opposite, across the diagonal. If you click and drag the phase of the z component, the reciprocating wedge pattern begins to rotate in the x/z dimension, to appear just like the simple rotation in Figure 3.8 when viewed from the top, while still appearing as a reciprocating wedge and anti-wedge in the x/y dimension like Figure 3.10 when viewed from the front.

Now click off the [0,0,1] node and replace it with the [0,0,2] node, second harmonic in *z*, along with [1,0,0] and [0,2,0]. This second harmonic in *z* oscillates in phase with the second harmonic in y, which converts the wedge and anti-wedge pattern into its three-dimensional generalization, a cone and anti-cone in the y/z dimensions as shown in Figure 3.11 B and C. As in its two-dimensional projected wedge form, this pattern morphs continuously from two parallel slabs in Figure 3.11 A, to a cone and anti-cone in Figure 2-18 B and C, to a cylinder and anti-cylinder in Figure 2-18 D, then on to another cone facing the other way, (E and F, not shown) to parallel slabs in opposite polarity (G, not shown in the figure) and back again through the reverse sequence again, and on to endless sequential alternation. The rotation of nodal surfaces we saw in Figure 2-17 is now no longer a simple rotation about two axes parallel to the z dimension, but a higher order

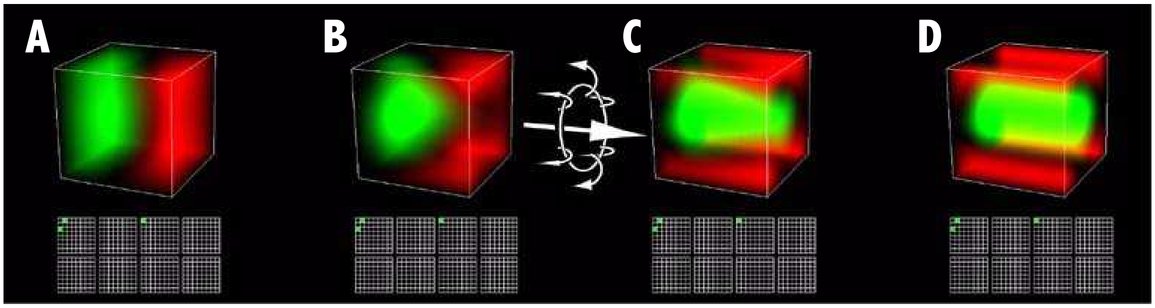

Figure 3.11. A: two parallel slabs, B: one bulges as the other shrinks back until C: the bulger punctures the shinker and D: turns into a positive cyllinder surrounded by negative field. The full pattern continues through three more stages in exact mirror symmetry to the first four, ending with parallel slabs in the opposite polarity. It then reverses direction and cycles through all the same patterns in exactly the reverse sequence each cycle, and repeats again for ever and ever.

rotation about a circular axis of rotation embedded in the z/y plane, as suggested in the overlay, with rotation first left-to-right through the center of the circle (the advance of the puncturing cone) with a reverse counterflow right-to-left around the outside of the circle (the counter-movement of the anti-cone) as suggested by the overlay, and a reverse flow in the second half-cycle to the left at the center, and to the right in the periphery (not shown). This pattern is suggestive of the most primal reciprocating motions of coitus, a motor pattern observed even in some of the simplest organisms.

We could go on endlessly with more examples of spatial, or spatiotemporal patterns and the harmonics required to generate them. But the significance of this concept is not so much in the details of the representation, or exactly how it breaks down shapes to express them in a sinusoidal basis set, but the true significance of harmonic resonance is that it demonstrates how an explicit, spatially-extended volumetric representation can be coupled to an abstract featural mechanism that is capable of both detecting patterns present in the representation bottom-up, as in visual recognition, as well as projecting patterns into the representation by reification, or filling-in, demonstrating the constructive, or generative perceptual function that is clearly a significant aspect of perception, and yet prominently absent from most models of perceptual representation and processing. And this explicit spatial representation is expressed in a holistic Gestalt-like mechanism that captures some of the observed field-like aspects of perception and conscious experience.

The effect of Resonator Shape

Although the cubical box of Falstad's Box Modes applet exhibits an extraordinary repetoire of complex patterns and combinatorial patterns, all of these patterns are related to the cubical form of the resonator itself. Even the complex tetrahedral patterns in Figure 3.6 are merely reflections of the cubical symmetry of the box, tracing the medial axis planes of symmetry between orthogonal faces of the cube. We see the same principle in the Chladni figures, that depend so much on shape of the resonating plate. Square plates produce rectagonal and diagonal patterns and sub-patterns, circular plates create radial and concentric patterns, triangular plates produce symmetrical partitions of the triangular form, as seen in Figure 1.16. And so also in three dimensions: a spherical resonator creates concentric, radial, and lateral type subdivisions, as seen in the atomic orbitals of Figure 1.18, whereas the cubical box creates the patterns shown in Falstad’s Box Modes applet. This strict dependence of the family of standing waves on the geometry of the resonator is useful for some applications, such as motor representations, where the shape of the resonator can be tuned to match the fixed topology of the body, but not so useful in others, as in perception, where the range of possible patterns or spatial interpretations of a stimulus should ideally remain as unrestricted and universal as possible. We will discuss later how this invariance aspect of perception can also be achieved in a harmonic resonance representation.

The usefulness of standing (and/or travelling) waves as a motor representation was discussed in chapter 1, with the example of a swimming eel, depicted in Figure 1.13, where the sinusoidal motor pattern propagating continuously from head towards tail, is modeled by a travelling wave in a cylindrical resonator, because a cylinder is a close enough model of the muscular topology of an eel. But where is this resonance located? Are we talking about a resonance in the eel’s brain and spinal cord, which is transmitted to the muscles by motor neurons? Or are we talking about a resonance in the muscle tissue of the eel, waves of electrochemical polarization and depolarization that create waves of contraction and extension in bulk muscle tissue? The example of coordinated motor patterns in simple creatures lacking a central nervous system, and of the cardiac muscle that continues to pump even after the cardiac nerve has been severed, demonstrate that it is *possible* for coherent waves of muscular contraction to emerge spontaneously in the absence of stimulation from motor neurons and a central nervous system, and it is unlikely that this basic property was lost when the central nervous system first evolved. More likely, the central nervous system evolved so as to make use of this more primal basic principle of spatial representation already inherent in muscle tissue. The central nervous system spurs the muscles into activity like a cowboy spurring his horse to stimulate a gallop, but the orderly waves of periodic contraction and extension that propagate through the muscles of the galloping horse are not sculpted or reified by that sparse and punctate stimulus, nor by the punctate motor neuron activation that the spurs indirectly trigger, but rather they emerge spontaneously from natural oscillations within the horse's brain, nervous systems, and musculature acting in unison, in response to the sparse stimulus.

This concept finally offers an explanation for the relatively sparse distribution of motor neuron synapses across muscle tissue, or *innervation ratio*, which can often be several thousands of muscle fibers for each motor neuron. According to the conventional explanation, each individual motor neuron innervates a patch of muscle tissue through a set of motor synapses distributed across the patch, to form a "motor unit", and patterns of motor contraction are explained by patterns of firing of such motor units triggered by activation in their motor neurons. According to this explanation, a patch of muscle in the motor unit integrates the activation from all the synapses in the patch, and contracts in proportion to a kind of average of all these scattered stimuli. This is basically a top-down concept, with all motor commands arriving top-down from the motor cortex. According to the resonance model on the other hand, the motor units do not form or define the waveform of muscular contraction, they merely trigger and synchronize it. But the contractile waveform itself emerges from the muscle tissue in response to stimulation, as a spatial reification process that occurs down in the lowest level within the muscle itself. This distinction could be tested neurophysiologically, at least in principle, by severing individual motor neurons in the middle of a muscle, and observing whether contractions stimulated in adjacent regions propagate also across the denervated region.

If the contractile motor pattern is indeed sculpted primarily by natural endogenous resonances, then the most effective way to trigger and modulate or control those resonances in the muscles would be with resonances in the motor system tuned to lock into and synchronize with the resonances in the muscles. In other words, the central nervous system also operates by harmonic resonance, that is, by coherent patterns of standing waves and travelling waves throughout the brain and nervous system, that communicate synchronous oscillations across widely separated locations. Perhaps the most powerful aspect of a harmonic resonance model of nervous system function is the natural tendency of resonating systems to couple with each other to establish a global resonance that is tuned not only to the natural resonances of its component parts, but also to the resonance of the emergent whole created by the coupling of the individual parts, as we see in the coherent waving of cilia in the paramecium, and in the coupling of oscillators like adjacent pendulum clocks, and the synchronous oscillations across the molecules of a crystal oscillator. This is the *Gestalt* in Gestalt theory, the whole is more than the mere sum of its component parts. The natural unifying properties of harmonic resonance are responsible for the coherence and synchrony between the experience of colors, motions, and sounds in perception, the coherence and synchrony of body motions in locomotion, mating, and dance, and the coherence and synchrony of perceptual experience with motor function, as a dancer matches their movements to those they perceive in their dancing partner, or as musicians synchronize their playing motions to the sounds they are perceiving from the rest of the band. The powerful unifying and synchronizing force of harmonic resonance is ultimately responsible for unity of conscious experience.

Rotation Invariance in Perception

The strict dependence of a harmonic resonance system on the geometry of the resonator is a disadvantage in the case of perceptual or cognitive representation, whose spatial patterns should ideally be determined by the properties of the stimulus, rather than of the resonator, so as to be able to represent any arbitrary shape that might be perceived, or imagined in mental imagery. It turns out that harmonic resonance has some pretty unique invariance properties that are extremely useful for a perceptual representation. In Figure 2.6 A we introduced the mechanism of a cubical box, with speakers mounted on three orthogonal faces, connected to three signal generators, that produce the x, y, and z components of the standing wave in the box. This mechanism does not have rotation invariance, because it draws a distinction for example between a second harmonic in *x* and a second harmonic in *y*, even though they are the same pattern only presented at a different orientation. The cubical "bottle" on the other hand, shown in Figure 2.6 B, and its related rank of organ pipes in Figure 2.6 C, is significantly different in this regard. Blowing air across the mouth of this cubical bottle is most likely to produce vertical pressure fluctuations, and thus standing waves in the *y* dimension, because the mouth of the bottle is located at the top of the box, so the pressure tends to fluctuate in and out of it. But it is in the nature of resonance that in fact any of the harmonics of the bottle can emerge, with greater or lesser probability, if sufficiently energized with a high velocity air flow, like the different harmonics of a bugle that can be obtained by pursing the lips and blowing harder. And among those many possible harmonics are the second harmonic in *x, y,* and *z*, any of which can appear in the bottle, especially when encouraged by careful positioning of the air stream. And the vibrational frequency of each standing wave pattern depends on the pattern itself, not its orientation. Therefore the “detection” of the presence of this second harmonic pattern by a sympathetic resonance in the second harmonic organ pipe, is a detection that is invariant to the orientation of the wave, it is a rotation-invariant recognition of the standing wave pattern in the resonator, at least for the six canonical orientations of the cube.

A similar distinction holds for the microphone that is used to record or sample the standing wave, like the microphone in Figure 3.2 B. A simple diaphragm microphone records the transient pressure fluctuations across the opposite faces of its diaphragm. A standing wave in the *x* dimension creates pressure fluctuations only across the *x* direction, so a microphone diaphragm oriented parallel to the *x,y* or *x,z* planes, could not pick up the x component of the vibration at all, because pressure fluctuations on one side of the diaphragm are exactly balanced, or cancelled by identical fluctuations on the other side. A simple diaphragm microphone therefore is a directional detector. But the microphone can be easily modified to make it invariant to orientation by sealing the back side of the diaphragm in a housing that is connected either to an external pressure reference, or to a closed reservoir of compressible air, to provide a pressure reference independent of the fluctuating pressure at that point in the resonator. This kind of microphone is omni-directional, it will pick up acoustical vibrations across the *x, y,* and *z* dimensions, and in fact, most microphones are baffled in exactly this manner for this reason.

The issue of directional invariance applies equally to loudspeakers. When the diaphragm of a simple loudspeaker advances (and retreats) during vibration, it creates a pulse of higher (lower) pressure in front of the diaphragm and lower (higher) pressure behind it, two complementary waves that cancel each other destructively around the edges of the speaker, leaving only a directional vibration in front of the diaphragm, and its negative mirror image behind it. As with the microphone, the speaker can be made omni-directional by sealing the back side of the diaphragm and connecting it to an independent external or internal reference, thus converting the alternating positive/negative pressure dipole of the directional speaker, to an effective “monopole” of alternating pressure at one location, that alternately injects and extracts pulses of air in the resonator, producing a spherically propagating wave from that source, expanding like rings in a pond. (The principal purpose of speaker boxes in audio systems is to seal off and baffle the negative wave from the back of the diaphragm, to prevent destructive interference with the positive wave from the front, and that is why speakers sound much quieter when removed from their speaker boxes.) The distinction between directional and non-directional detection (through microphones) and emission (through loudspeakers) bears on our model of resonance in the acoustical box.

If the resonator box filled with an array of glowing LED circuits depicted in Figure 3.2 A, were constructed using directional microphones, there would have to be three orthogonally-oriented microphones, with three amplifier circuits, at every location in the grid, and the oscillations would occur entirely independently across *x, y*, and *z* dimensions. But if the microphones are configured to record omnidirectionally, then they would detect sound waves in *x, y,* and *z* dimensions, and thus only one circuit would be required at each location, and the response of the system would become invariant to the orientation of the pattern in detection. It is the same circuits that resonate, for example, to a second harmonic wave pattern, whether oriented in the *x, y*, or *z* dimension. If an omni-directional resonator of this sort is connected to omni-directional detectors, like the organ pipes suggested in Figure 3.2 C, then the presence of a second harmonic of any orientation in the box would trigger a sympathetic resonance in the second harmonic organ pipe, and the resonance in the organ pipe would in turn amplify the corresponding waveform in the resonator.

Huygen’s Principle and Phased Array Antennas

A regular grid or lattice of identical loudspeakers, or other wave sources, can generate coherent globally patterned waveforms by Huygen’s principle, as seen in phased array antennas. According to Huygen’s principle, a wave front propagating through space is equivalent to a line of point sources, each propagating waves that radiate outward like the rings in a pond when a stone is thrown in. If a straight line of stones is dropped into a pond simultaneously, the outward-propagating rings from each individual source add together by constructive and destructive interference, summing together their common component, which is a wave front parallel to the line of sources propagating outward at right angles to the line, while all the other waves cancel each other by destructive interference. This too is a distinctly Gestalt phenomenon, where the global coherent configuration is preserved, while the local artifacts quickly fade due to lack of global support. This principle is employed in phased array antennas, where the phase of the signal in each individual antenna of the array is carefully tuned to produce the desired global waveform. For example if the antennas in an array are triggered to emit a train of pulses in uniform sequence from left to right, this will produce a moving wave, travelling left to right, at a speed determined by the time interval between successive pulses, which will leave a pair of angled wave fronts in its wake, like the bow waves of a boat moving left to right. If the antennas are triggered in two waves, one travelling left-to-right, one right-to-left, to meet at the center, this will create two sets of angled waves, like the bow waves of two motor boats that collide head-on in the middle. And if the speed of the propagating wave is varied as a quadratic function of distance from the center, starting faster near the periphery and slowing to some speed at the center at the collision point, this can create parabolic waves propagating outward as if from a parabolic dish, all from a flat antenna. This is the principle employed in modern phased array radars, in which a flat antenna composed of a grid of identical transmitter elements, generates a focused radar beam as if projected from a parabolic antenna dish, and the direction of the projected beam can be controlled on the fly by varying the phase of the signals radiating from one side of the antenna relative to the other, creating a fully steerable focused beam from a fixed flat antenna with no moving parts. This same principle allows the array of omni-directional loudspeakers in our acoustical box to generate virtually any waveform travelling in virtually any direction, by simply controlling the relative phase of transmission from the individual speakers. The phased array principle also operates in reverse, for a receiver antenna instead of a transmitter, that allows a fixed flat antenna to receive as if it were a steerable parabolic antenna, by simply time-delaying the signals from the individual receptor elements by the same pattern as that used in transmission. For focused detection of a distant source that is radiating waves outward in expanding shells, it is necessary to time-delay the signals from the center of the antenna relative to those in the periphery, so as to receive the spherical wave fronts simultaneously from the periphery and the center, and at every intermediate point, each delayed by the appropriate quadratic function, as if the detectors were mounted on a parabolic surface instead of a flat one. The quadratic time delay turns the flat antenna into a functionally parabolic one, and further time-delaying the signals coherently from left to right, or top to bottom, makes the antenna functionally equivalent to a steerable radar dish.

The phase patterns required for a phased array radar are computed on the fly by on-board digital hardware following mathematical formulae. How could these kinds of computations be performed in the wetware of the brain, or in the case of a simple acoustical box? The answer is to make use of the principles of harmonic resonance itself to perform the required computations to define coherent spatial patterns for a phased array transmission. A first harmonic standing wave, for example, vibrating in the acoustical box, creates a coherent wave front that travels back and forth in orderly fashion from one side of the box to the other, triggering the array of microphones arrayed within the box with exactly the right pattern of synchronized waveforms that correspond to the first harmonic resonance. For example if the audio signals from all of the microphones in the array throughout the box were recorded in a multi-track acoustical recording, then played back in the same box, with each track played back in the same location where it was recorded, this will re-create the original first harmonic standing wave back in the acoustical cavity. Furthermore, this re-creation is not just a top-down pattern imposed on the resonator, but rather, it is a re-creation of the original harmonic conditions that formed that first harmonic standing wave in the resonator in the first place. The wave emerges spontaneously in response to the trigger of the phased array pattern encoded in the loudspeaker signals. As in the case of the ennervation ratio of the motor units described above, the phased array signal need not recreate the whole waveform in all its sinusoidal detail in a totally top-down process, it merely needs to trigger the emergence of a first harmonic resonance in the resonator, and this can be achieved even with a very sparse “ennervation ratio” using only a handful of loudspeakers that capture a coarse sampling of the required waveform in the acoustical box, to recreate that wave back in the same box at the full original resolution. As that first harmonic emerges, it automatically sculpts itself to mathematical perfection by simple resonance in the resonator. All that is required of the “top-down” signal is to favor the first harmonic over other harmonic alternatives, the resonance itself will take care of filling-in or reifying the waveform to its maximal resolution.

This concept of phased array triggering of spatial standing waves offers a paradigm to explain the intimate coupling between spatial patterns in the different cortical and sub-cortical areas across the brain. For conceptual clarity, we have been discussing microphones and loudspeakers as distinct reception and emission devices. In fact, microphones and loudspeakers operate by exactly the same principle, a simple microphone will also serve as a loudspeaker, and vice-versa. For reciprocal coupling between cortical areas, it is more useful to think of the microphones and speakers in the acoustical box as one and the same element that can work in both directions, transducing vibrations of the diaphragm into electrical oscillations in a circuit, and transducing electrical oscillations in the circuit to vibrations of the diaphragm. Picture a pair of acoustical boxes, each equipped with an array of microphone/speakers, with a set of parallel connecting wires that connect each mike/speaker in one box to the mike/speaker at the corresponding location in the other box. A standing wave stimulated in either box will tend to stimulate the same standing wave in the other box, the two boxes will resonate together in a larger coupled resonance of both boxes simultaneously, in which any modulation of the resonance in one box will be communicated immediately and in parallel to the other box. This, I propose, is the principle by which standing wave patterns in one cortical, or sub-cortical area are transmitted to adjacent areas, and also how the retinal image is transmitted up the optic nerve to the brain.

Full Rotation Invariance with Spherical Resonator

The rotational invariance of the standing waves in a cubical box is a direct consequence of the number of rotational symmetries of the cube, that is, that the cube remains geometrically identical at six different orientations (12 different directions, if polarity is considered), and thus resonances at those orientations are essentially identical. An even more impressive degree of rotational invariance can be obtained by using a still more symmetrical resonator. If the cubical resonator is replaced with a spherical one, the family of standing wave patterns changes from the cubical repetoire to the spherical harmonic series, with its patterns of concentric shells, and periodic stripes like the lines of longitude and lattitude on a globe. Any of these spatial patterns can appear in the resonator at any orientation, while vibrating at a frequency that is characteristic to that waveform whatever the orientation, and thus the spherical resonator offers a completely rotation-invariant representation.

Invariance in recognition implies a many-to-one relation between the basic pattern and its many possible manifestations. For example a spherical harmonic, such as the third harmonic of the d-mode resonance shown in Figure 3.12 A, can appear at a range of orientations in three dimensions, while being the same essential pattern. A bank of tuned resonators, like the rank of organ pipes depicted in Figure 3.12 B, would respond to this characteristic pattern independent of its orientation through a full 360 degree rotation in any of three dimensions. But what about the top-down influence? If the organ pipe corresponding to this third harmonic pattern is energized top-down before there is any discernable pattern in the resonator, as suggested in Figure 3.12 C, which orientation will the pattern choose to appear in the resonator? If the top-down resonance can create a standing wave at any orientation, how would it choose one orientation to reify over all the other

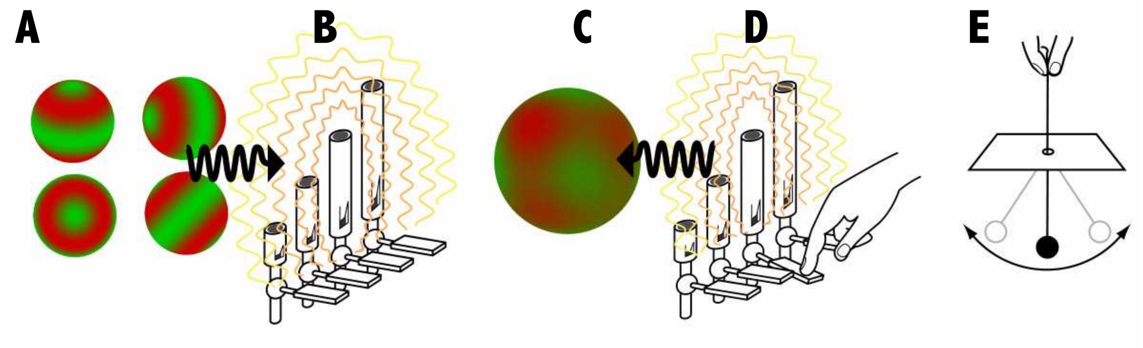

Figure 3.12. A: A third harmonic d-mode spherical harmonic resonance has a characteristic vibration frequency that is invariant to its orientation, and thus B: a bank of tuned resonators would serve as a rotation-invariant representation of that pattern. D: If that third harmonic is stimulated top-down, it will C: attempt to reify the corresponding standing wave pattern at all orientations simultaneous, producing an unstable indeterminate pattern, but if the priming is strong enough, it will select one orientation over the others, by the same principle that E: a pendulum's swinging can be amplified even if the orientation of its oscillations is unknown.

alternatives? The answer to this question turns out to involve one of the most interesting features of a resonance representation, the ability to perform top-down priming through an invariance relation. This principle will be explained with a simple analogy.

Imagine a pendulum on a string that is free to swing in both *x* and *y* dimensions, but whose support string passes through a small frictionless hole in a horizontal floor or divider between the pendulum bob and its point of support, as suggested in Figure 3.12 D. If the pendulum is set to swinging, the fact that it was swinging could be detected at the support point even if the bob could not be seen from above the floor, by a periodic variation of string tension due to centrifugal force as the pendulum swings, although it would be impossible to determine from above what direction the pendulum was swinging below the floor. The oscillating tension of the string can be seen as a rotation-invariant signal indicating the swinging of the pendulum at some orientation, independent of the orientation of that swinging. Is it possible to amplify this oscillation in the invariant representation above the dividing floor without knowing its orientation? How can the pendulum be given periodic pushes to keep it swinging in the direction it is already swinging, without knowing what that direction is? More generally, is it possible to provide top-down feedback and amplification across an invariance relation? In this case the answer is yes, by simple sympathetic resonance. If the pendulum is hanging motionless, it is impossible to pump it into swinging by gently pulling up and down vertically on the string's support point, the pendulum would simply remain vertical as it moves up and down. However if the pendulum already happens to be swinging even just a tiny bit in any direction, then synchronized pulling at the support point, timed to coincide with the periods of increased string tension, will amplify the swinging of the pendulum at whatever orientation it happens to be occuring, without any knowledge above the horizontal floor as to which direction of oscillation is being amplified. In fact, even if the pendulum were initially motionless, *vigorous* up-and-down oscillation of the support point would inject random oscillations which could subsequently be amplified by further synchronized oscillation. This is the same principle by which an invariant higher-level representation can serve to amplify its pattern through an invariance relation, amplifying preferentially the one oscillation orientation that is already the most active, at the expense of all the alternatives.

This principle will apply in the condition shown in Figure 3.12 C. At first, playing the third harmonic note top-down in the absence of any coherent pattern in the resonator, will attempt to activate the third harmonic standing wave at all orientations simultaneously, as suggested in Figure 3.12 C. If the top-down priming is weak, the system will remain in this indeterminate state, with a third harmonic oscillation at many different orientations simultaneously, as a fuzzy superposition of states. But if the amplitude of the priming is increased sufficiently, the pattern at some random orientation will happen to be stronger than the others, and as this occurs, a positive feedback loop with the top-down resonator will preferentially amplify that one orientation over all the others, a top-down priming with reification across an invariance relation. Furthermore, after the top-down priming has established a reified manifestation of its corresponding third harmonic pattern in the resonator, that pattern will remain free to rotate to any other orientation as long as the priming continues, because its orientation is not constrained by the priming, but remains a free variable. This is a very significant feature of a harmonic resonance representation, inherited directly from the physical phenomenon of harmonic resonance itself, that captures an essential aspect of perceptual recognition and reification.

For example we can easily recognize simple geometrical shapes such as cubes, rectangular blocks, pyramids, tetrahedra, etc. independent of their orientation; our recognition of simple forms is rotation invariant. But although our recognition is rotation invariant, it is not at all “blind” to orientation; we can easily see the orientation of the recognized object as soon as we recognize its characteristic form. We both perceive it, and perceive it to be at a particular orientation. Furthermore, we can easily reify the hidden rear faces of the recognized object, predicting the exact location and orientation of its hidden faces and vertices based on its visible portions, and we can even recognize and perceptually complete by reification objects which are partially occluded by foreground obstacles, even if the object is translating and rotating randomly through different orientations while we view it. This invariance in perceptual recognition, but *specificity* in perceptual reification, has been one of the most persistent unsolved riddles of perceptual function.

The principle of this invariance can be easily demonstrated using a Chladni plate. As in the case of a spherical resonator, a standing wave pattern on a circular Chladni plate can appear at any orientation, and each standing wave pattern will vibrate at its own characteristic frequency. An audio recording of this resonance, when played back in the presence of the circular plate, will tend to recreate the original standing wave pattern back on the plate, but the pattern can appear at any orientation. If the top-down priming is sufficiently strong, then one orientation will emerge as the dominant one, and amplify itself at the expense of the alternatives by positive feedback with the invariant resonance.

Rotation invariance is a very powerful feature of a harmonic resonance representation, as it helps to resolve the combinatorial explosion that would arise if every variation of the pattern (in this case every orientation) required a separate and distinct pattern template (in this case standing wave) to either detect or regenerate that pattern. Even more impressive invariances can be achieved by using a *gradient refractive index*.

Gradient Refractive Index

Besides the geometrical shape of a resonator, another factor that influences the shapes of the standing waves in a resonator is the *gradient refractive index profile*, as it is called in optics, a coherent modulation of the speed of wave propagation through the medium to vary continuously as a gradient across the resonator. Falstad’s Box Modes applet assumes a uniform speed of wave propagation, or speed of sound throughout the resonator, as is typical of an acoustical system. If the resonator is composed of biological tissue, however, this need not be the case; it is possible to create resonating systems in which the speed of wave propagation (or refractive index) varies signficantly across the resonator, according to some regular function, as in the example of *gradient refractive index* (GRIN) optics. Figure 3.13 A shows a GRIN lens in the shape of a cylinder whose refractive index varies continuously as a function of radial distance from the cylindrical axis, with greater refractive index toward the center, indicated by the shading. Typically a parabolic or exponential gradient refractive index profile is used, as shown in Figure 3.13 B. This has the effect of bending light in

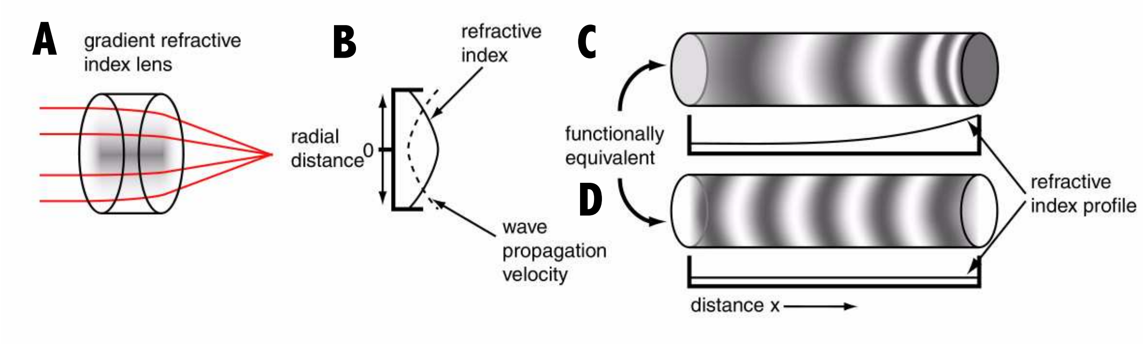

Figure 3.13. A: A gradient refractive index (GRIN) lens whose refractive index varies B: as a function of distance from its cylindrical axis with a parabolic or exponential gradient refractive index profile. C: If a cylindrical resonator with a nonlinear GRIN profile is used as a resonator, the standing waves that it sustains will be correspondingly distorted by the GRIN, although the resonance will be functionally equivalent to D: an undistorted resonance in a resonator with a constant, or uniform GRIN profile.

the direction of the greater refractive index, allowing a cylinder of glass with plane end-faces to behave like a convex lens, bending a parallel beam of light to a focal point. A reverse gradient with lower refractive index at the center than the periphery, would behave like a concave lens that makes a parallel beam of light diverge. A gradient refractive index in a resonator, for example using a GRIN lens as a lasing cavity, would also distort any standing wave that it sustains. Figure 3.13 C shows a standing wave along a cylindrical resonator whose gradient refractive index profile varies along the length of the cylinder, with lower refractive index and thus greater propagation velocity, to the left, increasing in nonlinear fashion towards the right. The standing waves in that resonator would be distorted by this change in the propagation velocity in such a way that the shorter waves towards the right in the figure oscillate back and forth at the same temporal frequency as the longer waves towards the left, and thus, all the waves oscillate in perfect synchrony along the length of the resonator dispite their differences in size. This squashed non-periodic pattern is thus *functionally equivalent* to a much longer un-squashed waveform in a uniform resonator with the same number of peaks and troughs, as suggested in Figure 1.13 D.

The possibility of imposing a regular distortion on the shape of the standing wave in a non-uniform resonator opens a unique opportunity to circumvent the boundary conditions at the boundaries of a resonator, and thus liberate the resonances within from the strict constraints of the geometry of the resonator. This in turn offers an opportunity to create resonances that are not strictly constrained by the geometry of the resonator’s bounding limits. The refractive index of a wave propagating material is a consequence of a time delay in the propagation. In the case of light travelling through glass, the time delay is due to the absorption, and subsequent re-emission of each photon of light many times as it passes through the material, effectively increasing the light path length to include a few orbits around each atomic nucleus that it encounters along its path. This slow-down in light propagation refracts a beam of light by the same principle that a column of marching soliers is deflected if it enters obliquely into a region of increased walking resistance, a ploughed field or low brush, which slows the column on one side before the other, deflecting its path toward the denser medium. In theory there is no limit to the magnitude of refractive index, up to an infinite refractive index, which describes a perfectly opaque material (light goes in but never comes out). In fact light itself has even been slowed down to bicycle speed in this manner, by using a Bose-Einstein condensate to delay the light in random fluctuations back and forth within the condensate before eventual re-emission. In a biological medium, whether chemical reaction-diffusion, or electrochemical oscillations, it is perfectly possible to create a system with very large, up to infinite refractive index using less exotic means than a Bose-Einstein condensate. In a theoretical model this is easily achieved by simply adding the appropriate time delay to the wave propagation equation.

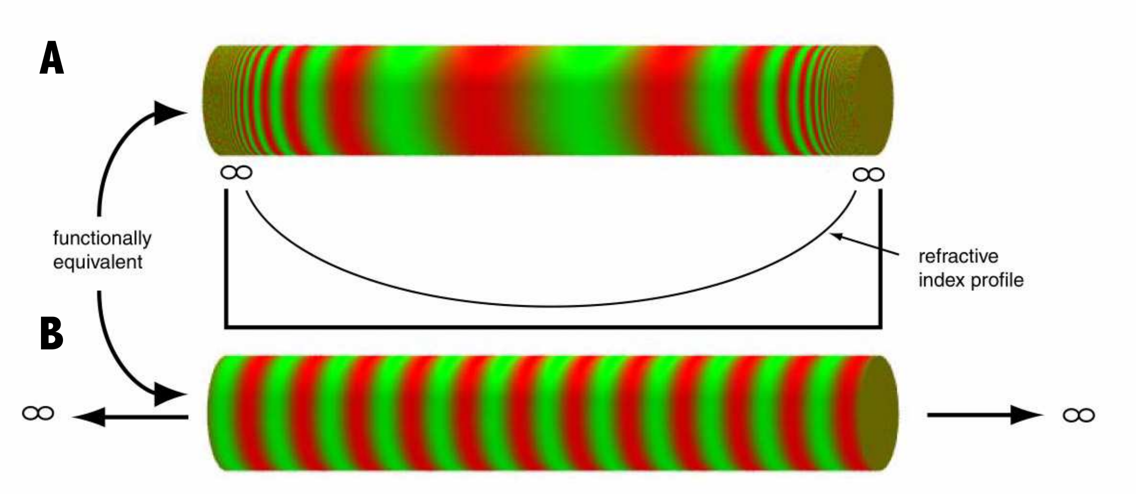

Figure 3.14. A: A resonator whose GRIN profile reaches infinity at the ends, will sustain standing waves with an “infinite” number of nodes towards the ends, which is functionally equivalent to B: a resonance in a uniform resonator of infinite extent, which is a physical impossibility.

Figure 3.14 A shows a cylindrical resonator with a rather extreme gradient refractive index profile that is relatively low at the center (high propagation velocity) but rises to an infinite refractive index at the two ends, or zero propagation velocity. Actually, it is not necessary for the refractive index to be truly infinite, it is sufficient to make it just very large, which is tantamount to “practically infinite” within some temporal window either side of the center. A wave propagating from the center outwards towards the ends, would propagate ever slower as it approached the ends, but never quite get there, (or only after a very long time) successive wave fronts piling up on each other like traffic slowing for a traffic jam. It is perfectly possible to set up self-amplifying standing waves in a resonator with this peculiar property. But since the refractive index is practically infinite at the ends of the resonator, the resonance is functionally equivalent (to some approximation) to an infinite resonance in an infinitely long resonator, as suggested in Figure 3.14 B. The use of a practically infinite refractive index, which is a physical *possibility*, allows the construction of something that is functionally equivalent to an amplified resonance in an infinitely long resonator, with an infinite number of nodes along its infinite length, something that is physically *im*possible. Of course the “trick” here is that there are not really an *infinite* number of nodes in the standing wave, there are nodes that get ever smaller and more compressed towards the ends, but in a real physical system this would only occur to some smallest scale, beyond which there would be no more vibrational nodes. Nevertheless, this “trick” has the effect of removing the effect of the abrupt bounding limits of the resonator, in a pattern that emulates the kinds of resonances that could emerge in an infinitely long resonator, but expressed in a finite bounded representation.

The standing waves in a normal finite length resonator are constrained by the boundary conditions at the ends. If the ends of a tube are closed, then vibrational nodes will form at the ends due to reflections back from the closed end. The waveform in the resonator is phase-locked to its ends. In the functionally infinite resonator depicted in Figure 3.13, on the other hand, there is no discrete end point to the resonator, and thus the wave that emerges spontaneously in a self-amplifying resonance is not constrained to a single phase, but can appear at any phase. In other words, the wave pattern shown in Figure 3.14 A, which exhibits an asymmetrical *sine* function across the center, with negative to the immediate left of center and positive to the right, could be phase-shifted 90 degrees to a *cosine* function, positive across the center and negative to the immediate right and the left, or it could be shifted to any intermediate phase value, while remaining balanced in resonance terms. That is, each of those phase-shifted waveforms would be equally valid or sustainable as standing waves in that resonator, and thus this resonator has become invariant to the phase of the waveforms that it sustains. It is a phase-invariant representation of waveforms in that space, in the sense that a tuned resonator tuned to the frequency of that wave would resonate to the wave regardless of its phase. This phase invariance also confers a frequency invariance to the standing waves that can appear in the tube. In a finite and bounded resonating cylinder with uniform refractive index profile, the standing waves that can be sustained are restricted to the frequency of the fundamental, and its infinite series of higher harmonics. The infinite bounded resonator, on the other hand, can sustain waveforms of any frequency equally, (within some lower and higher limits) and thus this resonator is both invariant to frequency and phase of the waveforms that it can sustain in self-amplified resonance. The resonator has broken free of the boundary conditions imposed by the discrete ends of the resonating system. The effect of the gradient refractive index is to insulate the resonances at the center from the boundary conditions at the ends, in the same way that a window function, like a *hanning window,* is used in Fourier filtering operations in image processing in order to eliminate the “ringing” due to the abrupt boundaries of the image, allowing a finite sized image to be treated as a finite sample of an infinite repeating pattern. This principle could be easily demonstrated by standing wave vibrations in a “gradient refractive index” slinky that is devised to have progressively increasing mass and stiffness towards one end of the spring, which is then fixed to a rigid attachment point. Normally, the waves that travel to an attachment point are reflected back in mirror-fashion from the abrupt transition of stiffness at the attachment point. In the gradient refractive index slinky, the transition is no longer abrupt, but continuous, and thus a wave propagating towards the end is reflected back continuously as it traverses the region of increasing refractive index, thus distributing the reflection process throughout a continuous region of the spring, and thus liberating the vibrations of the spring from the strict constraints of the abrupt boundary. The consequences of this principle extended into two- or three-dimensional resonances are even more dramatic.

The Infinite Bounded Sphere

The concept of a functionally infinite resonator can be extended into two or three dimensions by creating a circular, or spherical resonator with a gradient refractive index that goes to infinity at the bounding limits of the resonator. Figure 3.15 A shows a standing wave resonance in a resonator with this “infinite bounded” property. This figure can be viewed as either a flat circular disk, like a circular Chladni plate with a gradient refractive index profile, or as a central great circle slice through a three-dimensional spherical resonator with a three-dimensional gradient refractive index profile. In either case, the speed of wave propagation varies continuously with distance from the center such that it takes equal time for a wave propagating outward from the center to cross each of the concentric shells

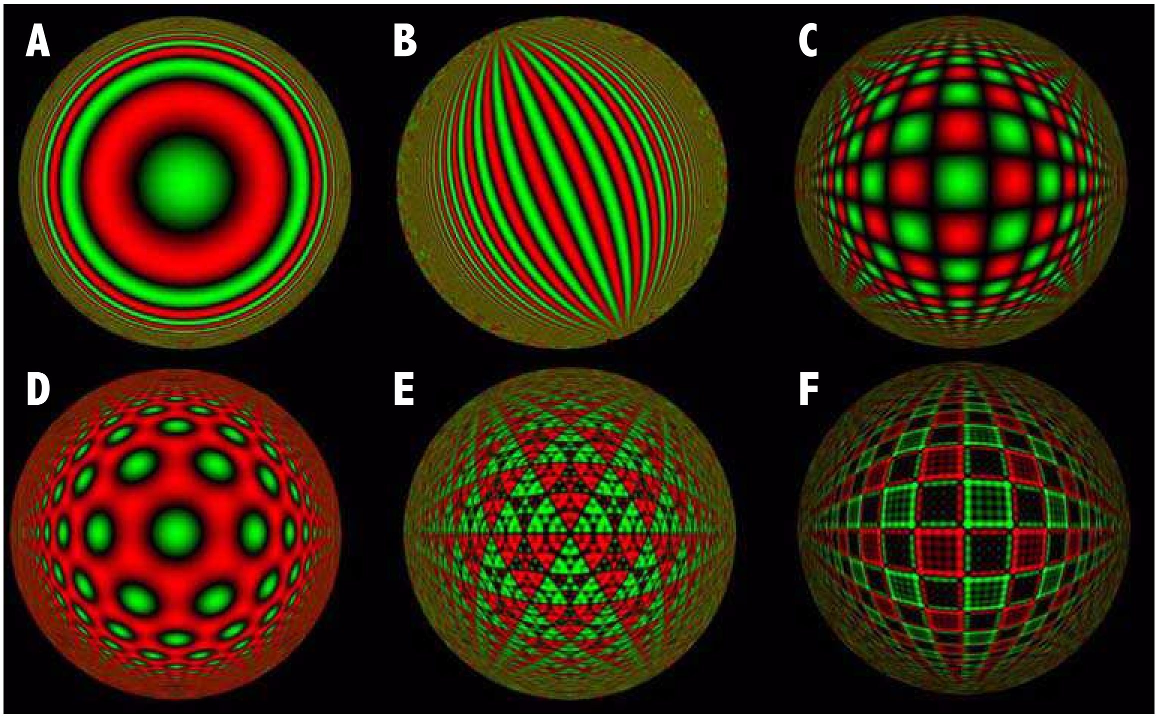

Figure 3.15. Standing wave resonance patterns viewed in cross-section of an infinite bounded sphere in which the refractive index varies with distance from the center, reaching infinite refractive index (zero wave propagation velocity) at the bounding surface of the sphere. A: Concentric pattern. B: Linear perspective pattern. C: Checkerboard pattern. D: Hexagonal pattern. E: Higher harmonics on a hexagonal pattern. F: Higher harmonics on a checkerboard pattern.

delimited by the depicted waveform, travelling faster across the rings near the center, and ever slower towards the periphery, slowing to a complete stop just as it reaches the outer boundary. This confers an additional “momentum” or functional inertia to the standing waves out toward the periphery, that allows a thinner compressed ring of the standing wave toward the periphery to resonate in balance with a thicker expanded ring nearer the center, as if they were waves of exactly the same size and shape balanced symmetrically against each other in a uniform resonator. As in the case of the GRIN resonator depicted in Figure 3.13 E, this resonator can sustain standing waves of any frequency within a range of frequencies between some lower and upper limits, and of any phase shift. But the full circular, or spherical closure of the infinite bounded sphere confers yet another invariance to this representation, an invariance to the shape of the resonator, a property that is highly desirable in a perceptual representation. Figure 3.15 B shows another standing wave resonance that would be sustained in the infinite bounded resonator, in this case a pattern of parallel sinusoidal waves at some orientation. Again, the smaller waves toward the periphery remain in dynamic balance with the larger waves through the center, but there is now a spatial distortion imposed on the standing waves, due to the redefinition of distance as a function of wave propagation velocity, that bends the parallel wave fronts into graceful arcs that meet at a pair of “vanishing points” in opposite directions across the resonator. In other words, this standing wave resonates in the same pattern as a field of straight parallel waves that extend to infinity in all directions, except expressed in a finite bounded representation. This resonator is now totally invariant to the shape of the standing waves that it sustains, because they no longer need to vibrate in balance with respect to the reflective boundaries of the resonator, but rather, the waves that emerge in this resonator must merely be balanced with respect to themselves, balancing the positive portions of the wave pattern against equal and opposite negative portions of the wave, a resonance of the similarity of the wave to itself, in either simple, or compound hierarchical form, rather than to the shape of the resonator. This representation now encodes two completely different kinds of spaces, all in a single representation. One is the spherical bounded and finite space of the resonator mechanism itself, that has a finite size, the other is a functionally infinite space (expressed to a finite resolution) in which distance is defined not by physical distance, but by the time taken to traverse it at at the speed of wave propagation through the medium. The world defined by the waves themselves is an undistorted Euclidean world of practically infinite extent, although with a finite spatial resolution that falls off to zero at the periphery. It turns out that this infinite bounded world has the same geometrical distortions as the world of our perceptual experience.

Figure 3.15 C shows another standing wave that would be sustained in the infinite bounded resonator, this one composed of two sets of parallel sinusoids oriented at right angles to each other, creating a checkerboard pattern of waves, that are also symmetrically balanced against each other, with exactly the same magnitude of positive and negative values across the pattern of a whole. Figure 3.15 D shows a hexagonal grid defined by three sets of parallel waves oriented at 120 degrees to each other, again maintaining a perfect balance of the wave against itself. These patterns could be sustained at any frequency through a range of frequencies, at any phase, and at any orientation through the resonator. The infinite bounded resonator is not confined to waves of sinusoidal form, but it can also resonate to patterns composed of fundamental wave forms and an orderly series of higher harmonics, to define patterns of arbitrary complexity. Figure 3.15 E and F show just two of an infinite number of possible combinations of wave patterns and their higher harmonics, with the higher harmonics expressed as integer multiples of some selected fundamental frequency representing the base pattern. Each one of the infinite array of possible patterns that can be composed of periodic patterns of sinusoids and their higher harmonics, are the natural modes of resonance of the infinite bounded resonator. That is, they express standing waves that would be sustained by self-amplification in the resonator if that pattern were imposed as an initial condition. Unlike the resonance of Falstad's box mode applet, this resonator is tuned to resonate to any pattern that has the self-similar property of either symmetry, or periodicity, or both, in some combination.

If a resonator of this sort is associated with a bank of tuned resonators, as suggested in Chapter 2 with the analogy of a bank of organ pipes, the tuned resonators would respond to their characteristic waveforms independent of rotation and translation of the pattern, and also to some extend invariant to spatial scale. For example the periodic striped pattern in Figure 3.14 B, oscillates at a single frequency characteristic of the scale of the waveform because the broad waves through the center vibrate at exactly the same frequency as the narrower waves in the periphery. A similar pattern of parallel waves that are either broader or narrower as they pass through the center, would produce a different characteristic vibration frequency that is either lower or higher in temporal frequency respectively. In other words, the temporal frequency represents the absolute scale of the pattern, even though the scale also varies from the center to the periphery. This is the property of harmonic resonance that accounts for the extraordinary invariance in visual perception and recognition to rotation, translation, and scale, as well as for the distortions of visual perspective, that have been so resistant to explanation in more conventional neural network terms.

Chapter 4

Harmonic Resonance in the Brain

We have seen that harmonic resonance exhibits some extraordinary and unique properties as a potential computational and representational mechanism even in a simple acoustical box, capable of generating reified spatiotemporal patterns that are causally coupled to a resonance mechanism that encodes those same patterns in an abstract symbolic code, using a Fourier type sinusoidal basis set. In Chapter 1 we saw that harmonic resonance is exploited by nature in embryological morphogenesis, as a spatial patterning mechanism for defining and reifying the spatial patterns of the body. But is harmonic resonance also exploited by nature as a computational and representational principle in the brain? In this chapter we will discuss the evidence for harmonic resonance in the brain, where it performs the same kind of function that it does in morphogenesis, and how it evolved from similar resonances in simple nervous systems, tracing back to resonances in some of the simplest creatures on the evolutionary scale.

Ascending the Evolutionary Ladder

The ultimate source of the electrochemical resonances in the brain and nervous system can be traced back to the properties of the animal cell. The earliest pre-cellular forms of life likely consisted of a loose confederation of chemical processes interacting in intricate feedback loops in rock pools or thermal vents. Eventually, a cell membrane evolved to prevent the vital components from simply drifting away into the primeval soup, to enclose the essential components in a semi-permeable bag, that self-assembles out of its component lipid molecules, which are manufactured by the molecular machinery of life for that purpose. The cell wall necessarily has to allow essential nutrients to permeate into, and waste products to permeate out of the cell, while containing the larger protein and genetic molecules essential for the biological machinery. But with the emergence of a cell wall, there immediately arose a problem, because the intra-cellular fluid has a high molarity due to all the heavy molecules of life that are dissolved in it, and thus osmosis creates a constant flow of water into the cell from the outside, that threatens to swell the cell and burst the plasma membrane. Bacteria and plants solve this problem by brute force, by making a polysaccharide cell wall that is tough enough to withstand the osmotic pressure, which can amount to tens of atmospheres, creating rigid cells with walls as tough as a pneumatic tire. Animal cells, in contrast, are in osmotic equilibrium with their environment and thus they are much more soft and flexible, and can take on a variety of different shapes that can change from moment to moment, like an amoeba. In animals the osmotic balance is maintained by a trans-membrane protein molecule known as a *sodium pump*, that is constantly pumping sodium ions out of the cell, against a concentration gradient, which also draws water out of the cell, and this maintains a constant electrical voltage, or *membrane potential*, across the cell wall in the order of 60 -100 mv, negative inside the cell and positive outside it, like a charged capacitor. The energy required to pump the sodium ions against the electrochemical gradient, and this energy is provided by ATP, the fuel used in living cells. The sodium pump is just one of a number of trans-membrane proteins that automatically embed themselves in the cell wall, that can open or close like a valve, allowing a flow of ions into or out of the cell, but preventing the escape of large protein molecules from the cell.

Many animal cells are endowed with voltage-gated channels that will open for a brief period, whenever the membrane potential drops below some threshold voltage, allowing a free flow of positive ions back into the cell. Neurons and muscle cells possess greater concentrations of these voltage-gated channels in their cell walls. This has a destabilizing effect on the membrane potential, because a small drop of membrane potential at some local point in the membrane for any reason, will tend to spread, resulting in an abrupt and total collapse of the membrane potential across the whole cell, as more and more channels open along a spreading wave of depolarization. After a brief refractory period, the ion channels slam shut again, which allows the sodium pumps, which are running continuously, to instantly restore the membrane potential across the cell wall, which creates a wave of re-polarization as the cell returns to its normal electrical imbalance. If the conditions that triggered the first depolarization prevail, this can lead to a continuous burst of repeating depolarizations, or a train of spikes of a spiking neuron, or a continuous vibratory contraction, or tetanus, in muscle cells. Neurons (and many other cells) are also equipped with chemically gated channels, that open in response to some particular neurotransmitter molecule locking into place on them from the extra-cellular fluid, causing a local depolarization of the membrane at that point, and these chemically gated channels can provide the trigger that sets off the series of periodic depolarizations that constitute a spike train of the spiking neuron. Although the opening and closing of electrically gated channels is generally in synchrony with the spiking discharges that they trigger, studies of the micro-structure of the temporal pattern reveals a more complex picture. Patch clamp experiments that record the state of single channels in an isolated patch of the plasma membrane have revealed a stochastic, or chaotic noisy pattern to the opening and closing of the channels on a time scale that is very much faster than the frequency of the typical spike train, (Sakmann & Neher, 1995) and some researchers have found stochastic resonance as a collective property of ion channel assemblies in somewhat larger patches of cell membrane. (Schmidt et al. 2001).

Although the opening and closing of electrically gated channels seems to be a random or chaotic process, it is possible that the process is not really random after all, but only appears so because we have not yet identified the causal agent behind it. It could be that this rapid opening and closing of voltage gated channels is not actually random or chaotic, but it could plausibly be in synchrony with a tiny voltage oscillation, or ripple voltage, superimposed on the larger voltage fluctuations of the action potentials when they arise, which in turn represents the electrochemical standing wave at that location in the neural tissue. That is, perhaps the electrical-gated channel serves the purpose of the diode in Figure 3.2 B, that opens during the positive phase of the oscillation and closes during the negative phase. In the presence of a noisy stimulus, like the sounds in many natural environments, it too would follow a generally random or chaotic pattern of positive and negative polarity. The apparently chaotic pattern of the opening and closing of individual ion channels is just a reflection of the complexity of the waveform of the electrochemical standing wave at that location in the neural tissue. In all but the most simple sensorimotor conditions (such as during an epileptic seizure, or tetanus), the waveform is a summation of many waves through a range of frequencies and phases that only appear chaotic at an isolated sample point. Whe viewed in the aggregate, as a spatial whole, even a complex chaotic waveform can be represented by a series of harmonics, as in a Fourier transform. It is the spatiotemporal patterns of those resonances that carry the significant sensory and motor signals, not isolated point samples of those signals, and the spatial wholes are also the aspect of neurophysiological activity of which we are consciously aware.

Perhaps the spiking neuron serves the purpose of energization and amplification of an analog waveform, like the reed of a wind instrument, whose vibratory motions are energized by a stream of air, but the vibration is a process is not confined to the reed alone, but involves the acoustics of the instrument as a whole. The purpose of the spiking neuron is not to register or record the pattern of electrical voltages received through its receptive field, as suggested by a feature detection paradigm, but acts somewhat like an antenna that both recognizes, resonates to, and energizes a spatiotemporal standing wave in neural tissue.

The biochemistry of the animal cell at the level of ion channels and trans-membrane proteins is very much more complex and elaborate than is covered in this quick summary, with most channels specialized to channel only one type of ion into our out of the cell, some gated electrically, others chemically, in response to a staggering array of different specialized neurotransmitter substances that each act on only their own specific chemically gated channel, with either excitatory or inhibitory consequences. But whatever the complexity of the systems of chanels and their neurotransmitters, there is no question that the molecular machinery of a single cell has the capacity to generate electrical voltages across the cell membrane with its sodium pumps, and to release that voltage periodically in abrupt depolarizations with electrically gated channels. This is easily enough machinery for a harmonic resonance mechanism with the capability of establishing and maintaining self-sustaining electrochemical oscillations across the cell membrane. And indeed, many animal cells, not only neurons, exhibit all kinds of electrical oscillations in a great variety of frequencies and waveforms, as amplified vibrations powered by ATP. The animal cell fairly pulses with electrochemical energy in a process that consumes energy continuously. The brain hums continuously with what Pribram (1971) has called *graded potential oscillations* (analog voltage oscillations) even in the absence of the overt action potential of the spiking neuron. There is no shortage of plausible neurophysiological hardware available to the animal cell to account for a harmonic resonance even within a single isolated cell, once you understand the inherent computational and representational capacities of a harmonic resonance system.

The cilia of the paramecium reveal a coherent wave-like propagation of some kind of signal, most likely an electrochemical travelling wave, that synchronizes the waving of its innumerable cilia to a coherent motor plan. The synchronous waving of the cilia clearly indicate the presence of a harmonic resonance in the plasma membrane even at this tiny single-cellular scale. Other single-celled creatures use a flagellum for propulsion, a long whip-like tail like that of a spermatozoa, which is composed of bundles of long protein molecules called microtubules that shift, lengthwise against each other in synchrony to produce the contractions and extensions that travel down the length of the flagellum. The undulatory or spiral-rotary motion of this extraordinary molecular tail is also clearly some kind of travelling wave resonance, that synchronizes and coordinates the local deflections at each point along the wiggling tail through local interactions with their immediate neighboring regions, to a globally coherent spiral or wavey pattern. Like the spermatozoa, many of these flagellated creatures exhibit goal-directed behavior, an ability to steer themselves toward some attractive stimulus, or away from some aversive stimulus. In the case of the spermatozoa, it is a chemical gradient that points the way toward a fertile egg. The Euglena, a single-celled flagellated animal capable of photosynthesis, is attracted toward light, whose direction it must somehow sense, and that sensory signal somehow modulates the pattern of waving of its flagellar tail. So even at this tiny microscopic scale we see creatures that close the full sensory-motor loop, whose motor system clearly involves travelling-wave resonances at a tiny single-cellular scale. Exactly how the sensory receptors of these tiny organisms control their flagella to steer towards the goal is not yet understood in any detail. But what is clear, is that whatever that sensory mechanism might be, it must have the effect of being able to intelligently modulate the patterns of resonances in the propulsive undulations of the tail, a sensory mechanism with causal effects on the oscillations of a resonance based propulsive mechanism.

In fact, this buisness of controlling sustained resonances has been the age-old pursuit of musical instrument makers for thousands of years, to devise resonating acoustical systems whose resonances can be modulated or controlled on demand. In the case of wind instruments, like the flute or clarinet, open holes damp the oscillation at that point in the tube, short-circuiting the oscillations of the vibration, triggering the formation of a node of oscillation at that point, whereas in string instruments the damping is achieved by pressing the sting against the fretboard with a finger.

Even at this microscopic single-cellular scale, the problem of sensory integration arises. A chemically guided organism presumably samples the chemistry of its external environment as does any other animal cell, that is with specialized trans-membrane proteins that have a special lock-and-key relation to the particular chemicals to which they respond, and the objective of the organism is to steer in the direction with the higher concentration of that attractor chemical. Many single-celled paramecium-like creatures exhibit this kind of chemotaxis. But if the sensory system of a single cell is composed of an array of trans-membrane proteins distributed somewhat randomly across its cell wall, how is the organism supposed to sense the pattern of those individual stimuli, to modulate the propulsive undulations of the multiple cilia, or a single tail, to steer the organism in the required direction? How would that signal energize or modulate the wave-like propulsive undulations observed in cilia and flagellae? Harmonic resonance offers a plausible explanation for this phenomenon. Suppose that the waves of electrochemical energy that put the wiggle in the sperm's tail, begin not at the base of the tail, but pervade the cell body of the sperm cell, starting at the anterior pole opposite the base of the tail, as suggested in Figure Figure 4.1. A. Perhaps the anterior pole is richly endowed with electrically gated channels, and thus tends to go off spontaneously, triggering waves that propagate down the cell body and continue down the tail. All that a sensory system would have to do is to find a way to modulate the phase of that wave, to slow it down on the side of the organism toward the stimulus, and speed it up on the opposite side, as suggested in Figure 4.1 B (or the reverse, for aversive stimuli). This would produce an asymmetry in the oscillations of the tail, steering the tiny organism in a curve. And all that would be required to modulate this travelling wave oscillation is a change in the dynamic parameters of the electrical oscillation across the membrane, something that might be easily accomplished by the opening or closing of chemically gated channels, which could either accelerate or delay the time required for electrical repolarization after each action potential. So the sensory integration of the individual ion channels as each one detects individual molecules of its target chemical in the extracellular environment, is achieved by summing or pooling the effect of each brief opening and closing of the chemically gated channel, as the individual gulps of ions released into the cell build up into pools of ionization inside

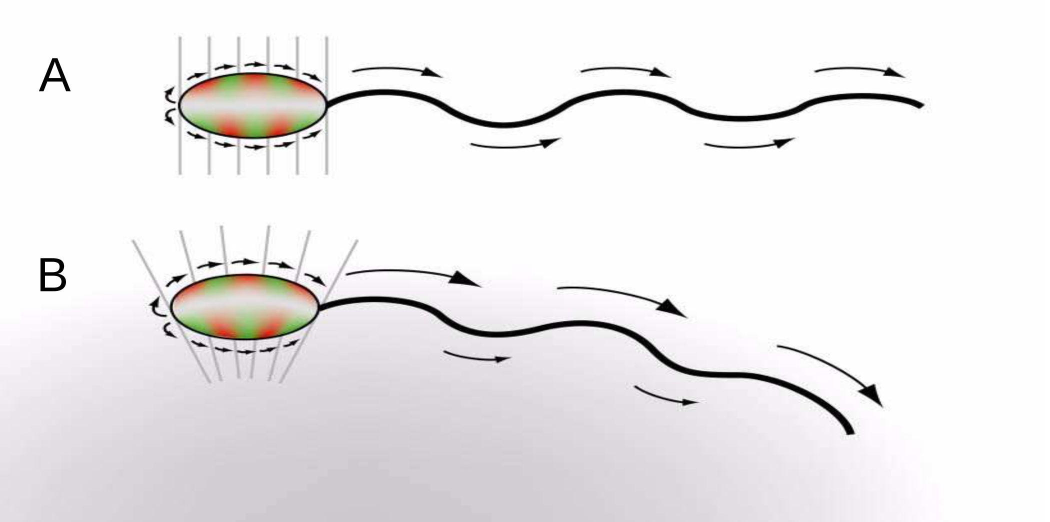

Figure 4.1. Wave model of sperm motor control, with A: waves triggered in the anterior pole of the cell body propagate to the posterior pole and beyond through the flagellum. B: The sensory system of the sperm has the effect of slowing the waves on the side of the organism closer to the stimulus, represented by gray shading, while speeding up the waves on the side away from the stimulus, resulting in a turning moment superimposed on the propulsive undulations.

the cell. If the ionic imbalance across the cell dissipates rapidly as soon as the stimulus is removed, that gives the ionic gradient the ability to respond rapidly to changes in the chemical gradients in the extracellular medium. And if the resonance is self-amplified, then even the subtlest enviromnental gradient would tip the balance of the intracellular resonances, breaking the symmetry between the otherwise equal balance of harmonics in different directions, like the opening of holes in a flute that select between discrete resonance patterns that pervade the body of the flute. The significant feature of this chemosensory system is not so much its absolute concentration, as much as the *gradient* of concentration across the cell. This gradient is copied, or mimicked by a resulting electrical gradient across the cell membrane, whose asymmetrical application across different sides of the microorganism produce a corresponding warp, or asymmetry in the resulting propulsive undulations. It is a spatial representation, expressed as a diffusion gradient across a volume of intracellular plasma, that controls a spatially reified spatiotemporal pattern of motor control. The computational problem of sensory integration is solved by an analog control system in a spatially reified replica, or model of external space in an internal representations. It is an analog modulation of an analog reified motor pattern, as analogical computations taking place across a spatial medium. And perhaps the most significant feature of this analogical principle of sensorimotor computation is that it automatically scales up from single-celled to multi-celled organisms with no additional hardware required.

From Single-cellular to Multi-Cellular Creatures

The transition of this resonance motor system from single-celled to multi-celled organisms can be tracked in an interesting family of transitional creatures known as the *Volvocales*, and their single-celled cousin the *Chlamidomonas*, shown in Figure 4.2. The Chlamidomonas, Figure 4.2 A, is a flagellated creature like the Euglena, that exhibits phototaxis, steering toward light in low-light conditions, and away from the light when it gets too intense. The Volvocales are composed of multiple cells stuck together, each cell equipped with its own pair of flagella, all the flagella directed outward away from the center of the cluster. Some Volvocales, like the Gonium, Figure 4.2 B, are composed of from four to sixteen cells, each one very similar in appearance to the single-celled Chlamydomonas, as if some ancestral Chlamydomonas had developed a mutation in which the cells fail to fully separate after mitosis. In another Volvocale, the Pandorina, Figure 4.2C, the cells form a more orderly sphere of 16 cells, and another, Eudorina, Figure 4.2 D, forms a sphere of 32 or 64 cells arranged in a perfect spherical pattern, rather than being just randomly stuck together, thus marking the first stage of embryological

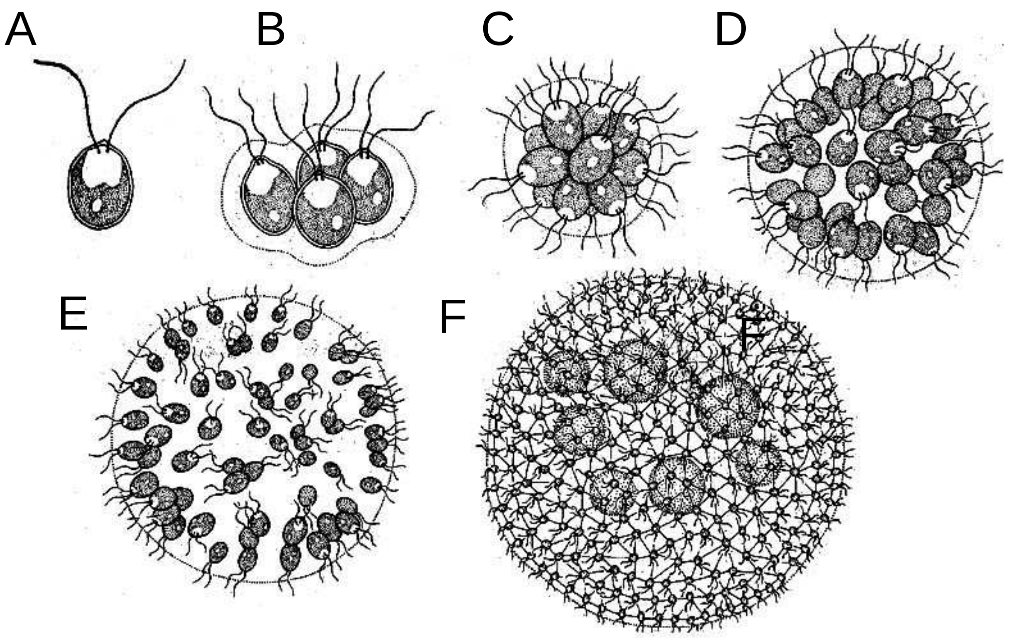

Figure 4.2. A: Single-celled animal Chlamydomonas, and a series of multi-cellular Volvocales with increasing numbers of cells; B: Gonium, C: Pandorina, D: Eudorina, E. Pleodorina, and F: Volvox. Although composed of a conglomeration of individual flagellated cells, the flagella of these compound organisms wave in synchrony as a single organism.

morphogenesis, the orderly division of one cell to produce a number of cells organized in a defined body plan. The next two genera of Volvocales, Pleodorina and Volvox, Figure 4.2 E and F, exhibit another advance into multi-cellular organisms, cell specialization. Whereas in Eudorina, every cell can, and usually does, produce a whole new organism by mitosis, in Pleodrina and Volvox, only relatively few cells on one side of the organism can reproduce, and in one species of Volvox, very few cells can reproduce, and those that can, never develop flagellae, having specialized for the role of reproduction exclusively. The Volvocales therefore exemplify the earliest stages of multi-cellular evolution, and they also illustrate the earliest stages of inter-cellular synchronization in sensory and motor function, because all of the volvocales exhibit the kind of phototaxic behavior performed by the single-celled Chlamydomonas.

Let us return to the Chlamydomonas. Unlike the Euglena, this creature actually has two flagella instead of just one, and they sprout from the anterior end of the organism, next to each other, and make mirror-symmetric waving motions somewhat like a swimmer’s breast-stroke, to promote locomotion in the anterior direction. So here we have two flagella that are physically separated, and yet act in symmetrical synchrony, obviously coupled, functionally speaking, through the cell membrane or cytoplasm, without any neurons or wires or mechanical connections, and that synchronous swimming pattern is responsive to the direction of light. (Cells of old cultures often exhibit disturbed synchrony between their flagella, with the result that swimming is erratic, cells frequently tumbling and turning uselessly in small circles, so the invisible synchronizing mechanism can be disrupted, with negative consequences for the organism.)

Now let us consider the ancestral Chlamydomonas whose failure to completely separate on mitosis founded the original Volvocale lineage. If the motion of its flagella are indeed synchronized by some kind of electrochemical oscillation through the cell membrane or cytoplasm, then this oscillation would also naturally tend to synchronize the electrochemical oscillations of the stuck-together cells. In other words, the very same mechanism that synchronizes the two flagella on a single Chlamydomona, could naturally extend to a synchronization of all the flagella on the compound organism, without having to evolve a new synchronization mechanism simultaneously with the spontaneous mutation that caused the failure to separate. The beauty of harmonic resonance is that it is so simple and primal of a process, that it naturally extends to oscillations in the compound creature composed of multiple cells, by the principle of entrainment of oscillations in similar oscillators. The remarkable aspect of Volvocale behavior is the perfect global synchrony between the motions of the flagella of its component cells, that somehow coordinate their direction of cyclic oscillation to conform to a global motor plan that allows the complete composite organism to behave as a single individual, that exhibits the same basic phototaxic behavior as the single-celled Chlamydomona. The coherence and synchrony of individual cells united into a single multi-cellular organism is seen most clearly in the Volvox, the most advanced of the Volvocales, a perfectly spherical organism made up of up to about 500 individual cells, arranged in geodesic symmetry, all of them synchronized and coordinated in the beating of their pairs of flagella, all in the absence of any kind of explicit synchronizing mechanism such as neural connections or mechanical linkages. Harmonic resonance is an extraordinary force for the emergence of global synchrony in identical resonators, at either the single-cellular, or multi-cellular level, and that is why nature makes use of this beautifully simple principle of harmonious interaction as the primal and primary principle of co-ordination and synchronization in organisms large and small.

The principle of harmonic resonance can also be invoked to explain the functioning of the simplest nervous systems in primitive creatures like the Hydra, discussed in Chapter 1. How could one possibly account for the structured and coordinated motor patterns of the Hydra, given its totally unstructured homogeneous network nervous system? How does the tissue of the hydra know where to contract and where to extend at each moment in time, when performing a synchronized behavioral pattern such as somersault locomotion? The resonances in the Volvocales, in the absence of any nervous system, demonstrate the principles of resonance in the simplest of animals. Surely the earliest nervous systems would build upon that more primitive resonance principle, rather than replace it with something completely different with the emergence of an explicit nervous system. The homogeneous fishnet type of network of the Hydra’s nervous system surely serves to amplify and modulate the more basic natural resonances of the Hydra’s tissue of animal cells, that provide the original sensory stimuli and actual muscular contractions. In other words, the Hydra’s network nervous system surely behaves as a resonator, like a musical instrument playing a sustained note, whose own natural resonances coordinate and modulate corresponding resonances in the tissue of the Hydra, and the pattern of those standing and/or travelling waves determines the patterns of posture and/or movement of the little creature.

Arthropod and Insect Locomotion

Further evidence in support of a harmonic resonance theory of neurocomputation can be found a little higher on the evolutionary ladder, in the patterns of locomotion of arthropods and insects. Millipedes and centipedes exhibit most clearly a wave-like pattern of motion of their legs, that surely must result from waves of electrochemical activity coursing down their spinal cords. As with the wave-like contractions of a swimming eel, the wave-like stepping pattern of a centipede is not just a simple wave, but one that is clearly modulated, to enable the creature to steer left or right, up or down, superimposed on the propulsive stepping undulations of its legs. And this walking pattern cannot simply be imposed top-down by a pattern generator in the creature’s “brain”, because the pattern of steps must adapt to the irregular surfaces and structures across which the creature crawls. Computing the appropriate leg deflection for each leg of the centipede as it walks and steers over irregular terrain while steering toward some sensory stimulus, poses a formidable control problem for any theory of sensorimotor function. A harmonic resonance mechanism naturally generates these kinds of wave-like patterns, and offers a simple mechanism for superimposing steering commands, and sensory feedback, on the propulsive undulations of a motor pattern generator, to define a single integrated sensorimotor plan of motor patterns modulated by sensory input.

Gallistel (1980) reviewed some of the theories that have been proposed to explain the wave-like motions of animals like centipedes and snakes and eels. Gallistel rejected the model of a simple reflex chain, where peaks of activation are passed along the spinal cord from anterior to posterior in sequence, like the progressive collapse of a line of dominos, because under ether, an earthworm contracts simultaneously along its length, which would be impossible for a reflex chain. Furthermore, an isolated nerve cord is observed to continue to oscillate electrically, like a snake that continues to writhe after it is chopped into pieces. Gallistel concludes that "*the nervous system is not, in fact, like a lazy donkey which must be struck ... every time before it can take a step. Instead, it is rather like a tempramental horse which needs the reins just as much as the whip.*"

The advantage of using a harmonic resonance principle for defining this motor plan is that it allows an immediate bi-directinal flow of information, both top-down from the "brain" to determine the direction and pace of the walking, but also bottom-up from each limb, to communicate its current posture back to the rest of the motor field. For example if one or more legs of the centipede are mutated, or injured, or temporarily bogged down, or snagged on some obstacle, the creature can react with a globally coherent and balanced motor response that takes account of that unplanned deviation from the standard walking template, while at the same time communicating the situation bottom-up back to the "brain" in case a change of plan is called for. In other words, the phase of the rotary wave that determines the phase of the stepping cycle for each leg, must be modulated not only top-down by a global motor pattern generator, but it must also be determined by somatosensory feedback from each leg, so that a leg that is bogged down and thus phase-delayed relative to the desired motor plan, will automatically lag the phase of its motor standing wave, which in turn will distort the whole global motor pattern, slowing down the phase of adjacent legs, which in turn slow down legs still farther away. In other words, the entire motor field is interconnected with bi-directional feedback forces like water seeking its own level in a closed vessel, every particle of water reacting continuously to any change in the quantity of water at every other point in the vessel

Amputation Experiments

The total interconnectedness of the motor patterns of the centipede has been revealed in a series of amputation experiments, where the removal of legs in various configurations leads to characteristic changes in the motor pattern of the remaining legs, in a manner that defies any simplistic explanation in terms of a reflex chain, or stereotyped motor pattern template, and strongly implicates a harmonic resonance principle of motor control. Gallistel (1980, p. 83-85) reports that the centipede Lithobius normally moves its legs in “wave” form, each leg separated from the next by a phase-lag of about one seventh of a step, so that each “wave” covers about 6-7 legs. If the centipede’s legs are amputated, leaving only six remaining legs, then the motor pattern adapts to this change of body plan by moving the remaining legs in the pattern of a six-legged insect, regardless of whether the remaining legs are contiguous, or separated by gaps of one, two, three, or more amputated legs in between. And if a further pair of legs is amputated, leaving only four, then the remaining legs exhibits the characteristic gaits of a four-legged animal, like a trotting horse. In other words, the more complex motor patterns of four and six legged locomotion are already pre-programmed into the motor pattern repetoire of the simplest of the arthropods, the centipede with its string of identical segments and simple wave-like walking pattern. Figure 4.3. is a copy of figure 4.6 (p. 84) from Von Holst (1980) .This is evidence of the *multipotential pattern formation principle* characteristic of harmonic resonance, in which a very simple mechanism is capable of not just one pattern of motion, but a whole array of different patterns that can be evoked under different circumstances. A similar adaptability in insect locomotion was demonstrated by Bethe (Gallistel 1980, p. 83), who showed that a six-legged stick insect whose middle pair of legs is amputated, demonstrated a four-legged gait like that of a trotting horse. This multipotential adaptibility of harmonic resonance is an essential prerequisite for evolution in the first place, if the organism is to survive and adapt to random mutations and variations in its body plan.

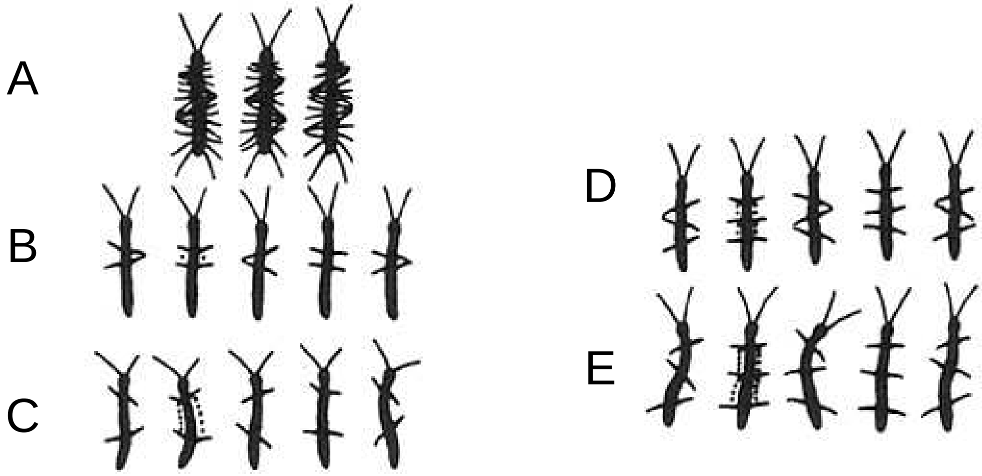

Figure 4.3. Phases from the running movement of the centipede *lithobuis* (from film records). A: Normal animal. B and C: after amputation of all pairs of legs except two. D and E: After amputation of all except three pairs of legs. (The dots in the second diagram in each series indicate the numbers of segments between the legs.)

In all these experiments a general rule was observed, that the smaller the number of legs present, the greater the phase difference between them. In more general terms this rule means that the processes in one ganglion are quantitatively dependent on the processes in all the other active ganglia. This interconnected coupling in turn implicates a harmonic resonance principle of locomotion.

Insect Locomotion

Arthropod evolution exhibits a progressive reduction in the number of legs, from hundreds in the milipede, and dozens in the centipede, to eight in the Arichnids, and to six in the insects, in each case adapting to the drastically altered body architecture with adaptive motor patterns that configure themselves to match the body to which they are attached. The centipede has basically only one gait, or pattern of stepping, and that is wave-like, although that wave pattern can be modulated in speed and direction, and even reversed for retreat. The six-legged insect, by contrast, exhibits an extraordinary six distinct *gaits* as it walks, as shown in Figure 4.4.. The legs of the insect are labeled as shown in Figure 4.4 A, and the plots on the left show the patterns of stepping motion of the insect's legs for the six gaits. The shaded rectangles represent times when a leg is in motion, swinging forward, the white rectangles represent times when the leg is in contact with the ground, pushing backward. On the right in the figure is the sequence of steps for each gait over time, to demonstrate the overlap between steps on the left and right sides. Not all insects exhibit all of these gaits, but examples of all these gaits are found across the insect world, and many insects are capable of several of these gaits. The gaits are listed in the approximate order they are observed based on the speed of locomotion, with the slower gaits at the top, and progressively faster gaits toward the bottom. The first gait, which might be called an *alternating wave* gait, is seen in slow browsing, with a wave of foot advancement from posterior to anterior on the right, alternating with a wave from anterior to posterior on the left. The next three gaits also exhibit posterior-to-anterior waves of stepping on left and right, with progressively increasing overlap or alternation between the left and right feet from Figure 4.4 B through D. These gaits are similar to the waves seen in centipede locomotion. Figures 4.4 B and C show how as the gait gets faster, the waves on the right and left sides no longer wait patiently until the other side finishes, but steps in overlapping waves on left and right.

Figure 4.4 E shows an interesting *alternating tripod* gait, that alternates between two stable tripod configurations with three legs on the ground, while the other

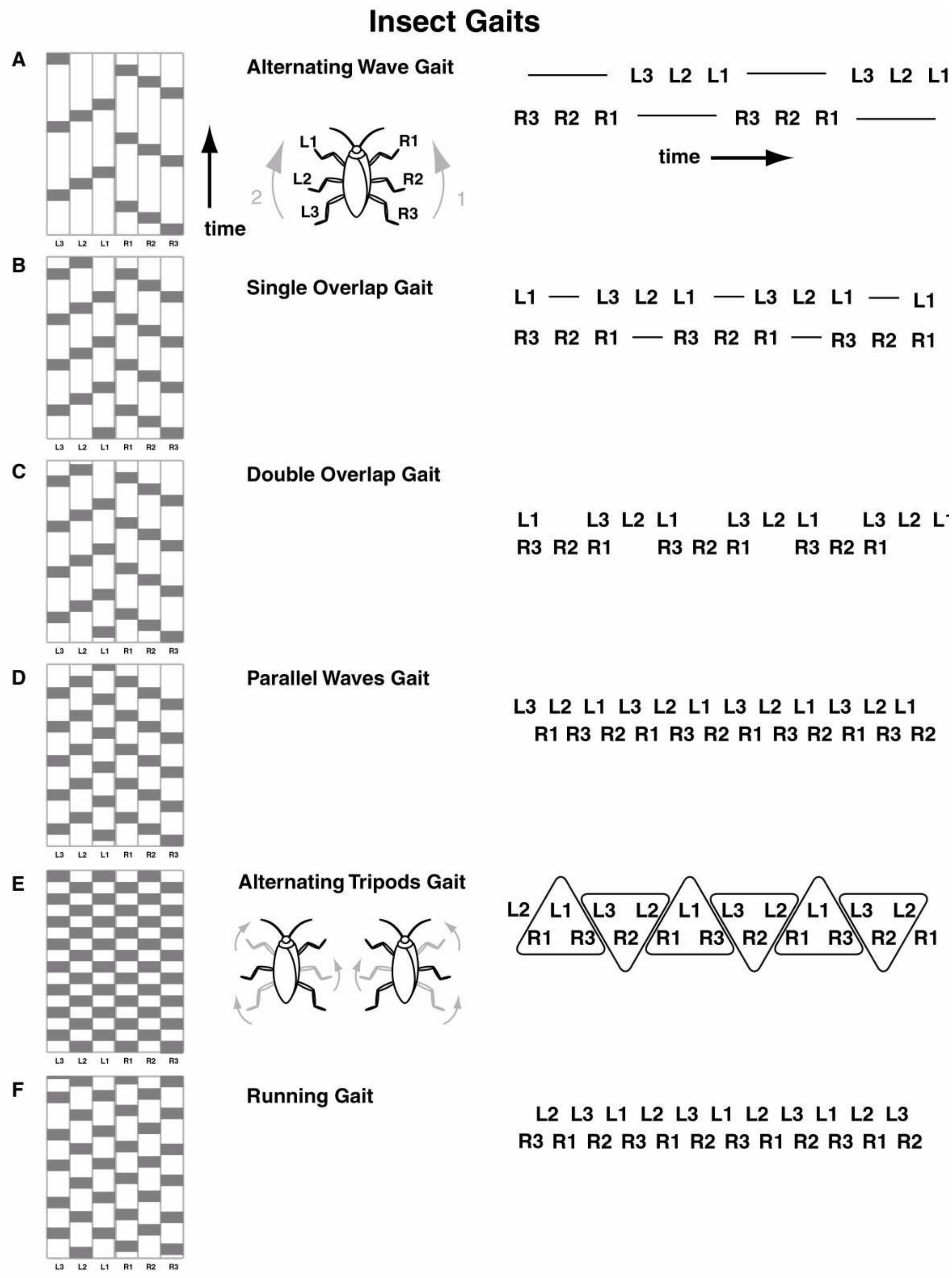

Figure 4.4. The six gaits of a six-legged insect. Most insects exhibit only a few of these gaits in their repetoire. The gaits are sorted by the order in which they tend to be used with increasing speed of locomotion, the early ones, A, B, C, are seen in slower movement, whereas the later ones, D, E, F, are seen in progressively faster running. The alternating tripods gaits, E, has a particular symmetry and is commonly found.

three legs swing forward ready for the next step. This is a particularly elegant and balanced gait, seen commonly in faster insects. And Figure 4.4 F shows a faster running gait in which the waves of leg motion exhibit a criss-cross diagonal striped pattern in the figure to the left, revealing interlocking waves of leg motions with two distinct components. Looking at the patterns on the right legs only in Figure 4.4 F, the diagonal patterns that progress upward to the left, (moving bottom to top in the figure) represent waves of leg motion from posterior toward anterior, like the pattern of Figure 4.4 A, whereas the diagonals progressing up and to the right represent waves of motion from anterior to posterior, interwoven with the waves in the opposite direction. The continuity of these diagonal rows of motion from the right to the left, and the other way, represent circular waves of leg motion that begin with a wave from posterior to anterior on the right side, flowing continuously into a wave from anterior to posterior on the left, that is, the waves travel up the right side and down the left, and round and round in circular repetition, while at the same time there are similar mirror reversed patterns going up the left side and down the right, like two counter-rotating circular waves that cause leg motions wherever they cross and thus interfere additively. These patterns are reminiscent of the kinds of rhythms seen in drumming, in which the left and right hand beats interlock in a variety of different patterns with abrupt transitions between discrete levels of interlocking patterns, which all exhibit the twin properties of symmetry and periodicity across tone and time.

The Gaits of a Horse

Evidence for a harmonic resonance principle of motor control is evident also in four-legged creatures like a horse. Figure 4.5.. shows the five gaits of a horse, or other four-legged creature, arranged, as before, in order of the speed of travel at which they typically appear. Figure 4.5 A shows the *walk*, which is similar to the insect walking pattern of Figure 4.4A, all the legs on the right side move from posterior to anterior, followed by the same sequence of the left legs. Figure 4.5 B shows the *trot*, an alternating gait in which diagonal pairs of legs are on the ground in alternating sequence. This is the four-legged equivalent of the insect’s alternating tripod gait. Figure 4.5 C shows the *pace*, in which the two left legs alternate with the two right legs. Figure 4.5 D shows the *canter*, a three-phase pattern in which a diagonal pair of legs (in this case the left front and right rear) step together, followed by each of the remaining legs stepping in sequence, in a “this - both - that, this - both - that” kind of sequence, a motor pattern with a Waltzing rhythm. There are two alternative forms of the canter, called “left lead” versus “right lead” canter, depending on which diagonal pair of legs steps together

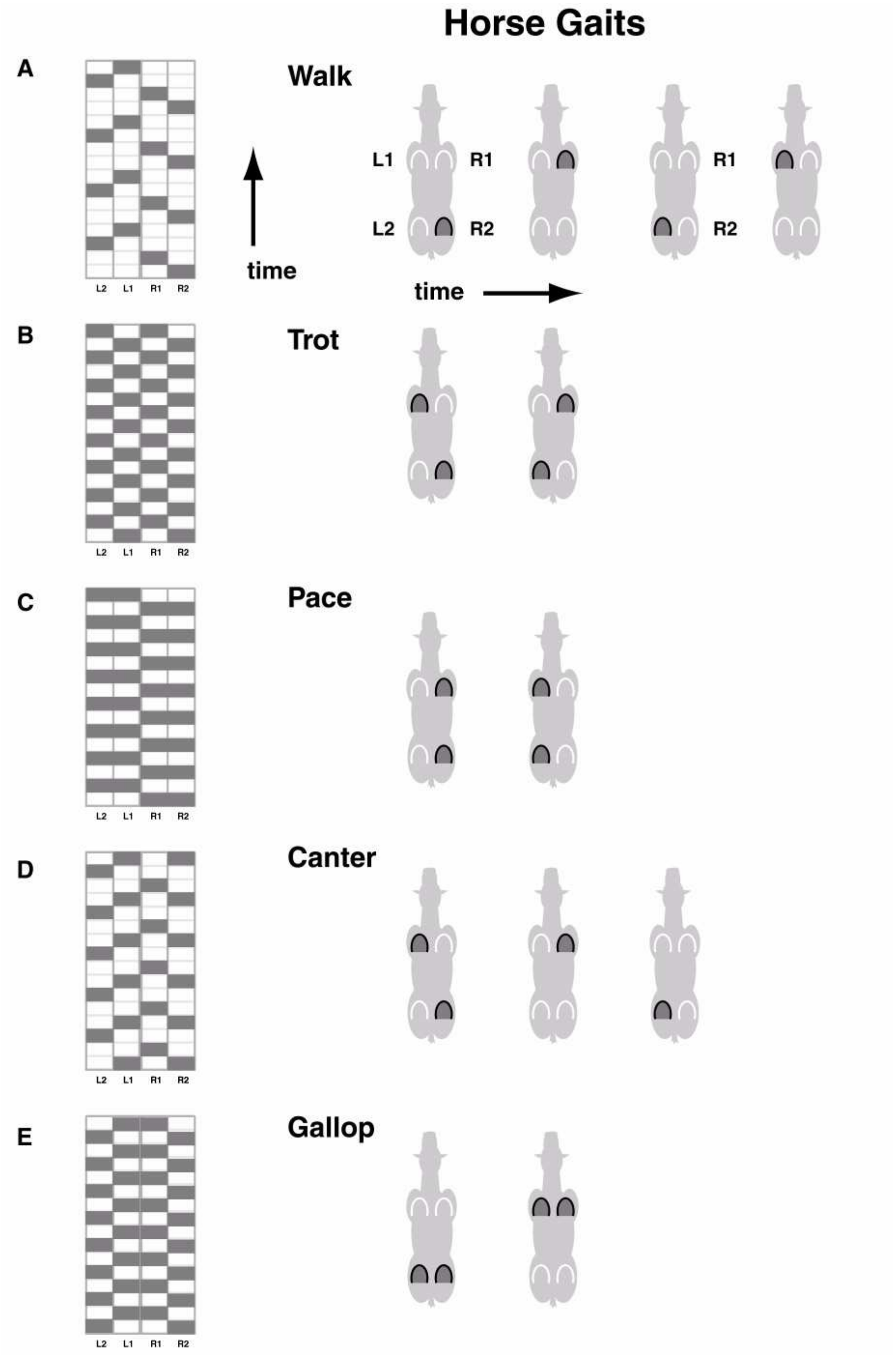

Figure 4.5. The gaits of a horse, plotted to the left, and illustrated to the right. The darkened hoofs represent the leg or legs that are stepping forward, the white horse-shoes represent legs standing on the ground.

and which step individually. Finally Figure 4.5 E shows the gallop, in which the rear legs alternate with the front legs in a two-phase front-and-back alternation.

Strogatz and Stewart (1993) observe that the patterns of gaits across different species with different numbers of legs, correspond exactly to the patterns of a set of coupled oscillators, with one oscillator for each leg. For example the four gaits of a horse correspond to the patterns exhibited by four coupled oscillators, whereas the six insect gaits correspond to the patterns of six coupled oscillators. According to this analysis, there is one additional gait of the four-legged animal which is not seen in the horse's gait repetoire, and that is the four-legged hop, known as a *pronk,* and sure enough, there are several species of antelope or ungulates that exhibit this gait also. This analysis also extends to two-legged creatures and bipedal gaits, which exhibit the modes of two coupled oscillators, walking and hopping.

The significant aspect of a harmonic resonance principle of motor control is that these various gaits need not be hard-wired in any kind of rigid template or schema for each gait, but rather, they are emergent properties of a motor system that is reciprocally inter-coupled, in such a way that the motion of each limb has an effect on the motion of all the other limbs, with the result that the limbs automatically interlock in periodic temporal patterns in which each leg moves once in each cycle, and different patterns emerge spontaneously under different walking conditions. When the urgency of locomotion, or desire to move, is slowly ramped up, the lowest energy patterns emerge first, then progressively higher energy patterns, with discrete steps between a fixed number of discrete modes. And yet, although each gait is a discrete pattern, it is also fundamentally analog, and thus easily modulated for the purpose of going faster or slower, and steering left or right, and since a harmonic resonance system automatically adapts itself from moment to moment to the current configuration of all the legs, this kind of system also automatically adapts to unpredictable circumstances in real-time, such as injury or fatigue to one or more limbs, or encountering unexpected or irregular resistance to leg motion. This is presumably why nature favored a harmonic resonance principle for locomotion over a more stereotyped cam or template-like schema.

Centralization and Modularization of the Nervous Systems

There is a clear trend in the course of evolution of the nervous system from simple distributed toward complex centralized nervous system architectures. The reason for this trend is seen in the simplest centralized nervous systems in the evolutionary successors to the hydra, the hydromedusae, the earliest progenitors of the squid / octopus line. These creatures combine a distributed network nervous system like the hydra's, with a fast-action giant neuron reflex system for rapid escape. Hydromedusae are like hydra that swim instead of remaining stuck to the bottom. They propel themselves through the water by squirting water from their mouth rhythmically, in a slow wave cyclical alternation of intake and exhaust during normal locomotion, or in powerful bursts for escape. The advantage of the centralized architecture is clearly evident in this creature. A noxious stimulus of sufficient magnitude from any point around the creature's body can trigger its emergency escape response, and yet that response, when fired, itself triggers a wave of muscular contraction, synchronized with a constriction of the mouth to increase the jet effect, alternating with an intake phase with mouth open. There is a many-to-one and one-to-many relation between the interaction of the peripheral and the central systems, which clearly exemplifies the principle of abstraction.

The nervous signal in the central system is very simple, just periodic pulses of electical energy in a linear resonator. But that signal is only useful or meaningful because of its ability to trigger travelling or standing waves of spatial pattern in the tissue, causing waves of muscular contraction in the pattern defined by the local phase. And the central system is useful or meaningful as a sensory system only because local sensations picked up by sensory organs anywhere on the creature's surface can potentially trigger a global escape response. The one-to-many, and many-to-one intercoupling between the central and peripheral systems is provided by spatiotemporal waves of oscillation in both sensory and motor representations.

The value of an abstract neural representation, like the one representing the escape response, is in direct relation to its efficiency in triggering that response, and thus it will have additional value if it can represent more than just a single stereotyped response, but pick from a wider repetoire of basic movements modulated by sensory input. It is also essential to keep the periodic pulses of the central system synchronized with the cycles of contraction of the musculature that it stimulates. If the contraction is slowed down for any reason, whether due to local injury or fatigue, it is best for the central nervous system to detect that response lag, and compensate with slower oscillations, or perhaps compensatory asymmetry in the motor pattern, so the central nervous system needs continuous feedback from the periphery to keep it in synch with external reality. This is where a harmonic resonance principle is so essential. Like a parent pushing a child on a swing, the pulses of thrust must remain in perfect synchrony with the swinging child, a task which humans find natural and intuitive, due to the resonance principle of our nervous systems. In fact, the very urge of children to swing rhythmically on swings and rocking chairs, is evidence of a harmonic resonance in their brains.

A general principle in the relation between the central and peripheral nervous systems activation can be characterized as follows. The central system emits a powerful pulse of electrical energy, then it waits in a refactory mode until it gets back an “echo” from the periphery acknowledging receipt of, and successful execution of the commanded contraction, before sending out the next pulse. There is a dynamic standing wave relation between the oscillations in each system, even though one is an abstract linear antenna, while the other is a volume of spontaneously resonating tissue in which spatiotemporal patterns are painted in three-dimensional space. This relationship is demonstrated in the principles of many musical instruments. For example the rude “raspberry” made by a trumpeter’s lips, acts as the central nervous system, its energetic vibration sets up the initial waves in the trumpet, but with the very first reflected wave coming back from the far end of the trumpet, a beautifully symmetrical and periodic standing wave establishes itself in the three-dimensional volume of the instrument, and a harmonious tone ensues, as the trumpeter’s lips are seduced into vibrating in synchrony with the perfectly periodic sound from the trumpet’s resonance.

Another significant property observed in the evolutionary development of central nervous systems is a *modularity*, or *multiplicative tendency*, a natural tendency to spontaneous form repeating copies of the same pattern elements. The principle is clearly evident in morphogenesis, as seen in the patterns of plant and animal forms, like sunflowers and starfish, fiddle-head ferns and centipedes. And the same kind of modularity is also in evidence in the periodic patterns of nodes, or ganglia, observed along the spinal cord in vertibrates. If the central nervous system of vertibrates operates on the principle of harmonic resonance, as I propose, then this periodic or modular architecture would have a natural tendency to resonate in modular standing waves that reflect the modular or multiplicative architecture of the nervous system itself. This principle of multiple similar resonances in a series of coupled, near-identical resonators, reveals a powerful organizational principle in nervous function that can be explored with a thought experiment.

The resonances in a simple linear spinal cord, i.e. a cylinder of spontaneously resonating tissue, without periodic ganglia would be similar to the resonances in a cylindrical cavity, the most basic of which subdivide the linear dimension of the resonating cavity into periodic intervals of alternating motion. The first harmonic resonance represents an alternation in time, with a wave travelling one way alternating with a wave travelling in the other, a back-and-forth reciprocal motion along the axis of the cylinder. The second harmonic is a 2x multiplication of that pattern in both space and time, with pairs of half-waves propagating from the ends to the middle, then reflecting off each other back out to the ends, two half-waves balanced against each other at double the frequency of the fundamental. The waves in this second harmonic mode are constrained to remain mirror-image replicas of each other, because if one half-wave were to be attenuated by some random resistance at some point in the resonator, then it would no longer have the momentum to completely reflect its counterpart, and thus its counterpart would also tend to lose that same component of motion when they collide at the center, automatically transfering energy from the stronger wave to the attenuated one, exactly in the place and time where that wave was lacking in energy.

This principle can be demonstrated in an acoustical model of the phenomenon. Consider a cylindrical flute, energized by air blowing across its mouth, capable of resonating at any frequency which is a harmonic of its fundamental, as suggested in Figure 4.6..A, showing the amplitude functions of the first six harmonic waveforms for that resonator (plotted in colors orange, blue, red, purple, pink, green). An open hole at the center of the tube, as suggested in Figure 4.6 B, would promote a node of vibration at that central point, because the open hole short-circuits any standing wave with an anti-node centered at that location, which

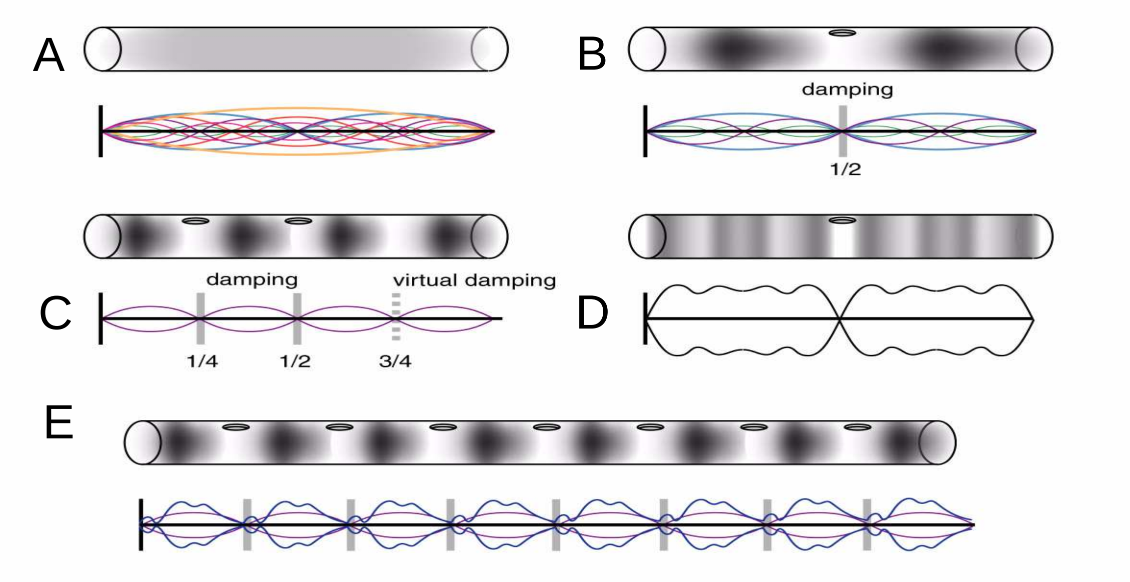

Figure 4.6. Modularization of the nervous system demonstrated with an acoustical analogy. A: A linear tube supports multiple resonances. B: An open hole at the half-way point damps the odd harmonics. C: A second hole at the one quarter position damps the second and sixth harmonics. D: Even more complex waveforms are constrained to mirror-symmetry across the central node. E: A flute with a periodic array of open holes is constrained to standing wave patterns that also repeat in periodic intervals, either a simple fundamental, (purple) or a more complex compound waveform (navy). But the waves are not merely dumb copies, they are causally interconnected to act as a whole.

includes the first harmonic, along with the third, and fifth, that is, the odd harmonics on the fundamental. With this open hole, the second harmonic resonance becomes the new fundamental of this now bimodal resonator, it is now the lowest energy note that emerges most readily from the resonator. This central node has turned the half-waves into mirror images of each other, reflecting back and forth across the central nodal point as if bouncing off a mirror in both directions. But these symmetrical half-waves are not merely similar in appearance, they are coupled in a relationship that constrains them to remain identical to each other energetically. For example if a second hole is opened at one quarter the length of the tube, as shown in Figure 4.6 C, this open hole will promptly damp any harmonic with a strong anti-node at that location, including the second harmonic (blue), which has an amplitude peak right at that spot, as well as the sixth harmonic (plotted in green) which is also damped. This damping affects the second harmonic wave not only there at the open hole, where the acoustical short-circuit is in effect, but it also damps its counterpart at three quarters the length of the tube, a mirror-image reflection of the node at the one quarter position. The hole is robbing energy from the resonance continuously, but robbing only energy at one specific frequency, the second harmonic, (and the sixth) and from two specific locations in the standing wave, at distances one quarter and three quarters. But since the energy leak is actually through the hole in the one quarter location, there is a continuous flow or leak of energy from the three quarters point in the tube, across to the other side of the center and out the hole at the one quarter point, because any spontaneous resonances that happen to emerge at this frequency will fail to be reflected back from the other half of the tube. The damping due to the hole at the one-quarter location creates a *virtual damping* at the mirror-symmetric three-quarters point, as if there were an open hole at that point also. A similar damping occurs with the sixth harmonic, whose damping at the one-quarter point effectively creates a weaker virtual damping at all the anti-nodes of the sixth harmonic in both halves of the tube, although the magnitude of this damping is considerably weaker than that of the second harmonic, because the open hole damps only one of the six anti-nodes of the sixth harmonic, whereas that same hole damps a full one-half of the two anti-nodes of the second harmonic.

The same principle applies also to more complex waveforms containing component higher harmonics, as suggested schematically in Figure 4.6 D. Even these more complex compound waveforms are constrained by the open hole at the center of the flute to patterns that are mirror-symmetric across the center, so that they can bounce off each other with perfect reciprocity. This is the principle of modularity, a characteristic feature or property of harmonic resonance.

Modularity is not restricted to the bimodal resonator split into equal halves by a central damping point, but the same principle can be generalized to a periodic array of holes at equal intervals, which also subdivide the resonator into equal resonators, which in turn would promote a fundamental frequency (purple in Figure 4.6 E) whose wavelength exactly matches the spacing of the holes, but with the same interconnected property of the bimodal resonance expanded to multimodal reciprocal reflections. Although each standing wave in each of the nodal intervals vibrates independently as if it were by itself in a single-antinode cavity, they are also subtly interconnected energetically, such that if some component frequencies are amplified or damped in any one of the identical resonators in the chain, there will be a continuous leak or flow of energy from the the other resonators toward the damped point, which in turn would establish virtual damping at the corresponding locations in their copies of the wave. The result is that whatever the waveform that emerges from this periodic array resonator, its waveform exhibits a strong tendency to be identical in each nodal interval, as suggested by the blue trace in Figure 4.6 E, and that waveform is shared among all of the resonators in a distributed fashion. Any additional damping that might occur in any of the segments, for example due to the opening of additional holes, would create a notch in the waveform corresponding to the location of that hole, and that notch due to damping in one segment is replicated in every other segment with a corresponding virtual damping. Thus the pattern of resonance in this periodic resonator is causally coupled across the array, each pattern intimately coupled with every other pattern in the array. This principle of modularity, or multiplicative tendency, is exploited by nature as a means of coupling, or synchronizing the patterns of representation in sensory and motor systems in a periodic array-like architecture of coupled resonators.

Beads On A Straw

The architecture of the vertibrate spinal cord is shaped not so much like a uniform cylinder, but more like a cylinder with periodic nodes, or ganglia, along its length, strung somewhat like beads on a straw, as suggested schematically in Figure 4.7 A. In a resonance neural architecture this configuration can be expected to establish nodes of resonance at periodic intervals to match this periodic architecture. But the prominent swellings at each ganglion promote a different kind of standing wave at those locations, more similar to the spherical resonances

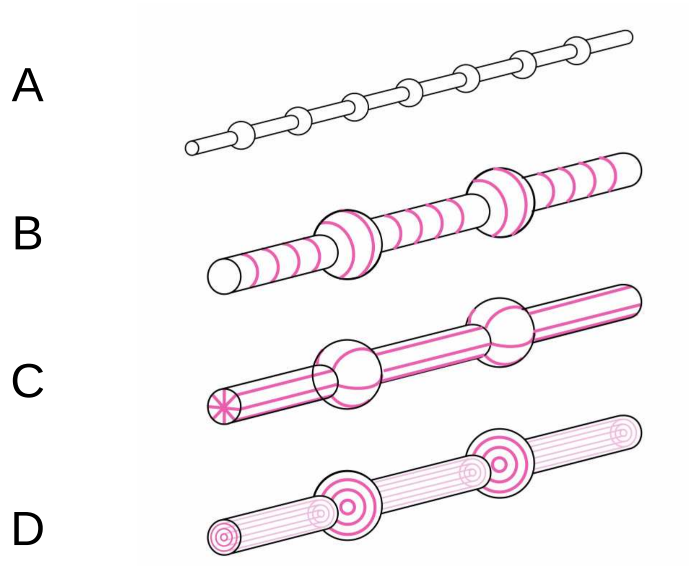

Figure 4.7. A: The topological archititecture of the vertibrate spinal cord is somewhat like beads threaded on a straw, a cylinder with periodic nodes, or ganglia. B through D: Some of the many possible standing wave patterns that would spontaneously emerge in a self-amplifying resonator that had this geometry.

of a spherical resonator, as opposed to the cylindrical resonances along the spinal cord itself. A spherical resonator has many more possible standing wave patterns than a cylinder, allowing for more possible patterns of sensory or motor encoding in each segment. Consider the basic modes of resonance of a spherical resonator, shown in Figure 1.18 A through D in Chapter 1. Each of these resonances could appear spontaneously in a self-amplifying spherical resonator, even one connected to other similar resonators by the resonating cylinder of the spinal cord. Some of the resonances in the spheres would couple with corresponding resonances along the spinal cord. For example Figure 4.7 B shows linear resonances back and forth along the spinal cord segments, interacting with similar one-dimensional lateral standing waves through the node, as if that node was part of the cylindrical spinal cord. Other modes of resonance are totally independent of, or invariant to linear resonances in the spinal cord. For example Figure 4.7 C shows radial resonances. These radial resonances occur orthogonal to the linear resonances, and thus would be entirely un-influenced by the linear resonances. Figure 4.7 D shows concentric, or “s-mode” standing waves in the spherical nodes, and there are additionally a number of rotational resonances with wave rotations in various directions. These additional modes of resonance offer additional degrees of freedom which can be employed for spatial representation in sensory and motor systems as explained below. Each of these standing wave resonance patterns would radiate its own characteristic wave outward into the surrounding tissue, where these periodic patterns are projected onto muscles that contract based on the phase of the wave that they detect.

Putting Resonances to Work

Consider the motor control challenge that arose during the evolutionary progression from a millipede, with large numbers of very tiny legs, toward a centipede with a smaller number of somewhat longer legs, to an insect with just six legs but each very long with multiple segments. As the legs get longer they must get less numerous in order to avoid collisions with adjacent legs. And they gain more degrees of freedom of motion, especially as they grow additional segments. The motor signals to a centipede's legs can be fairly simple, a wave of deflection not so different than the waving of the cilia of the paramecium. But the legs of the centipede each have two degrees of freedom, forward / backward and up / down, along with an "up-forward-down-back" propulsive cycle for locomotion, as shown in Figure 4.8 B. This additional degree of spatial encoding can be provided by spherical resonances that represent the walking cycle, while resonances along the spinal cord can serve to synchronize and coordinate the resonances in the individual ganglia. I cannot say exactly which of these almost infinite possible standing wave patterns are actually employed in the sensory and motor functions of vertibrates, but we can discuss some of the possible resonances that might be employed, to illustrate how resonances can serve to

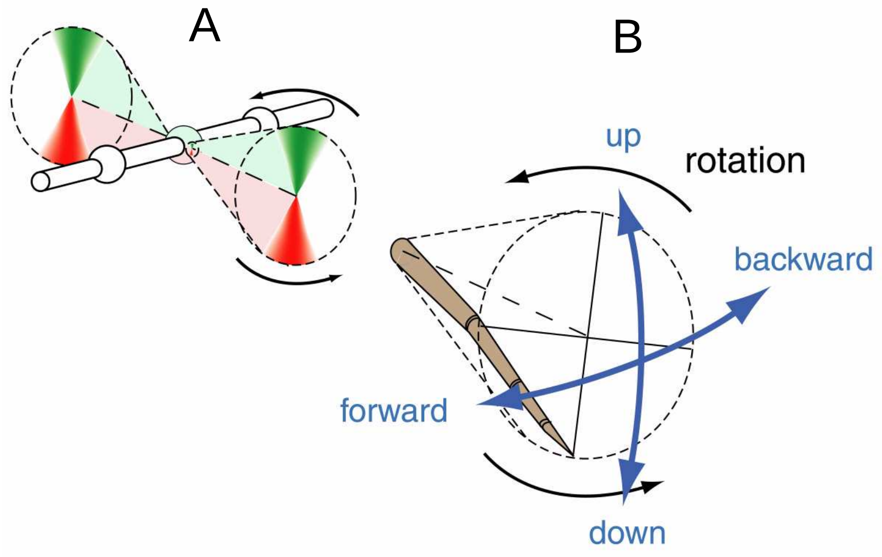

Figure 4.8 A: The degrees of freedom of a centipede's leg, forward, backward, up, and down, along with a rotational pattern used in locomotion, up-forward-down-back, can all be encoded with a rotating standing wave pattern in the ganglion, whose direction of rotational phase controls the angle of deflection of the leg.

express complex spatiotemporal patterns in a globally integrated dynamic mechanism.

Hypothetical Model of Vertibrate Motor Control

Figure 4.8 A shows a cyclic oscillation through a ganglion of the spinal cord, whose phase represents the current phase of the stepping cycle at that point in the spinal cord. The two dimensions of the stepping cycle are easily encoded in spherical harmonics, and the phase from node to node is communicated by travelling waves along the spinal cord, lawfully time-lagged from node to node. The spinal cord need only communicate one dimension of information, the phase of the stepping cycle, and that simple one-dimensional synchronizing signal can synchronize an array of more complex three-dimensional resonators in phase with each other’s patterns along the spinal cord. This kind of rotational standing wave resonance could explain the synchronized patterns of rotary motions of a millipede’s legs.

As the short simple legs of the millipede evolved into the longer compound legs of the centipede, the same resonance architecture can be adapted to express the more complex patterns of motion required to control the individual segments of each leg in synchronized motion. The extra dimension of radial distance along each leg, can be encoded by a radial pattern of standing waves in the central nervous system, like the s-mode standing wave resonances of a spherical resonator. The general principle here for expressing the additional dimension of control is the same as the principle outlined above, where the central nervous system emits an abrupt pulse of electrical activity which propagates outward in all directions peripherally, to be reflected back from the terminals of the nervous system back to the central nervous system. Like the principle of echolocation used by bats, this principle can be used to map out the peripheral nervous system, and to build an analogical mirror-image replica of it back in the central nervous system.

Figure 4.9 A. I shows a simplified model of the nervous system of a centipede with segmented legs, each ennervated by a central nerve running down that leg, with nodal points disposed at intervals along the leg, corresponding to the series of joints, each of which offers still more degrees of freedom of motion. Imagine a powerful pulse of nervous energy that begins at the junction point where the spinal ganglion meets the nerve from each leg. That pulse would propagate down the nerve of the leg, sending back echos, or reflections, as it passes each node along the leg, as suggested in Figure 4.9 C through E. A listener, located at the nerve

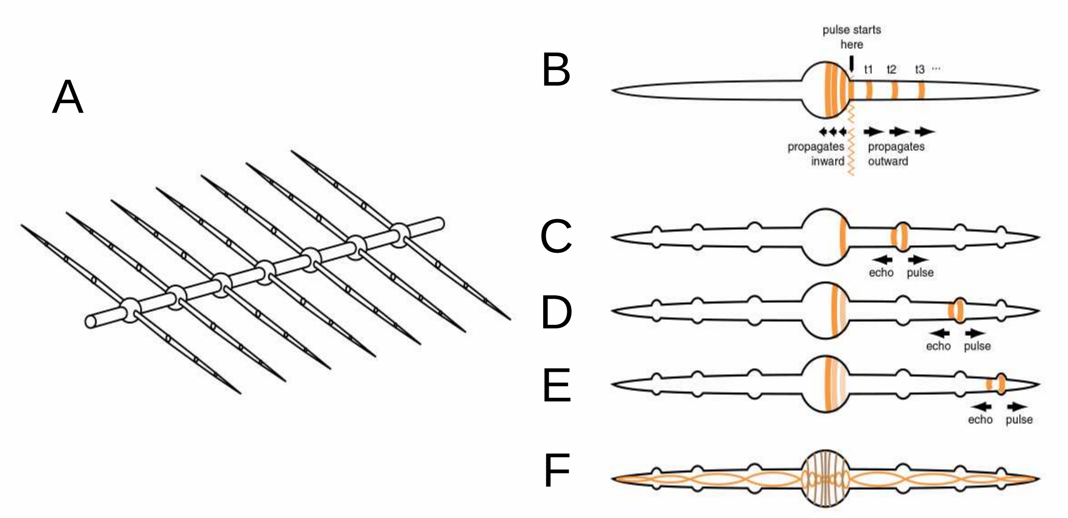

Figure 4.9 A: An idealized model of the centipede's nervous system, with nerves for each pair of legs projecting from the vertibral ganglia. B: A pulse of electrical energy starting at the nerve root, (C, D, E...) propagates outwards as a centrifugal wave, and inwards as a centripetal wave, sending back echos as it passes through the nodes. F: Cyclic pulses establish coupled, mirror-image reflected standing waves in the nerve and the ganglion.

root, listening to the pattern of echos returning from the leg, could determine the architecture of the segmented nerve, and thus of its segmented leg, by the segmented pattern of returning echos.

Now imagine that the centrifugal pulse that propagates from the root of the nerve to the tip, is matched by a centripetal pulse travelling in the opposite direction, from the nerve root towards the center of the spherical resonance of the ganglion, as shown in Figure 4.9 B, but with a considerably slower propagation rate, so that the centripetal wave reaches the center of the ganglion just as the centrifugal wave reaches the tip of the limb nerve. What we have is two mirror-image waves sweeping outward from the nerve root in synchrony. If the speed of wave propagation is very much slower in the ganglion than it is along the nerve, then these two mirror-image waves are presented at different spatiotemporal scales, and yet they remain in dynamic balance against each other. The centripetal sweep is a miniature copy of the centrifugal one, and if the speed of propagation declines in nonlinear fashion toward the center of the ganglion, that would define a nonlinear shrinking spatial scale as discussed in the previous chapter. The spatiotemporal similarity of these two waves in different media offers a means for plotting the spatial pattern of the limb as a corresponding spatial pattern in the ganglion.

The nerve that runs along the leg has little nodes or ganglia at each segment, that are like a miniature model of the ganglia along the spinal cord, and operate by similar principles. As the centrifugal impulse passes through these nodes, it sends back a powerful echo that records the pattern of nodes in the limb nerve. As the pattern of periodic echos returns from the limb, those echos can be plotted in the central nervous system at the location of the centripetal wave, at the time of the returning echo, just as returning echos are plotted on a radar scope by an electron beam that sweeps in a pattern similar to that of the projected radar beam in external space. The periodic echos in time plot a spatial pattern of traces disposed radially in the ganglion, corresponding to the radial pattern of nodes along the legs. Now if the centrifugal pulses are repeated at intervals timed to the return of the last echo, this will establish a pattern of radial standing waves in spherical shells in the spinal ganglion, as suggested in Figure 4.9 F, to match dynamically similar patterns of standing waves along the nerve of the leg. This establishes a map, or representational framework coupled with the standing waves in the nerve that it models, at a scale that is proportional to the difference in propagation velocity, and this map can be used to encode the posture or motion of the leg.

Each joint of the leg is equipped with proprioceptive sensors that sense the degree of flexion or extension of the joint, in two dimensions at each moment of time. Now let us imagine that the signal from these deflection sensors is used to accelerate or retard the speed of propagation of the centrifugal waves through the nerve at that location, by an amount proportional to the degree of deflection. This would serve to deflect the direction of propagation of the waves travelling along the nerve, retarding them on the side where contraction is sensed, and advancing them on the other side, as we saw in the case of the spermatozoa model of Figure 4.1, and this has the effect of bending the whole wave in the nerve into a curve. Since the resonances in the nerve are coupled with those in the ganglion, this deflection in turn would distort the standing wave pattern in the ganglion too, providing an explicit spatial model of the posture of the limb at any time.

This principle is demonstrated in Figure 4.10 showing the distortions of the standing waves in a peripheral nerve when its limb is deflected, and how that distortion is reflected in a corresponding distortion of the standing waves in the spinal cord. Bending the whole leg coherently forward, bends the standing wave uniformly at each node, thus warping the whole standing wave pattern as it resonates in the nerve, sending asymmetrical return echo pulses back to the ganglion, where those pulses paint out a correspondingly distorted standing wave that mirrors the pattern of deflection of each of the joints in the limb. An analogical spatial representation for the purpose of performing spatial computations in the

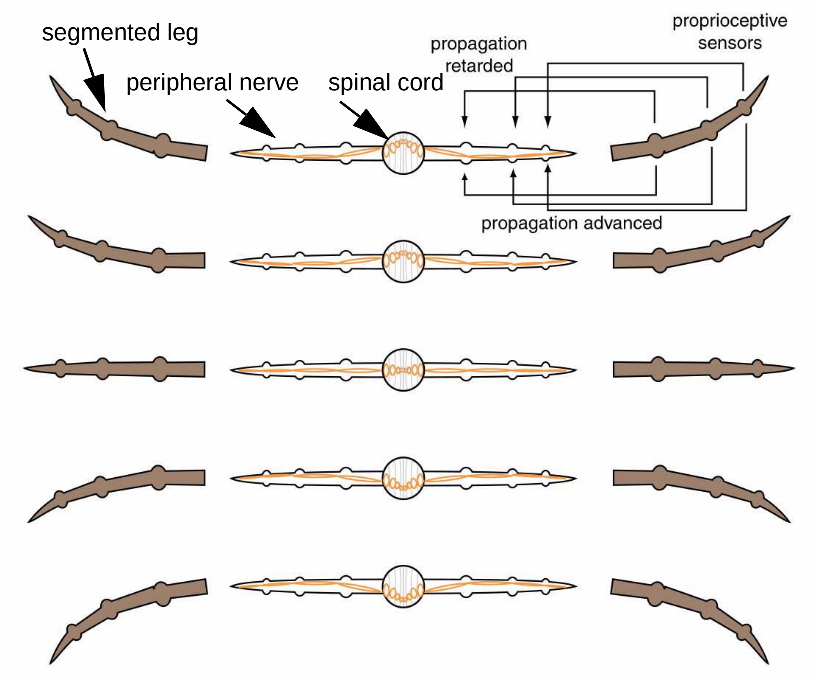

Figure 4.10. Proprioceptive sensors in each joint of the limb transform sensations of extension and compression to signals that advance and retard the speed of wave propagation in the local part of the nerve. A coherent distortion of the whole limb bends the standing wave in the nerve, and this in turn bends its mirror-image counterpart in the ganglion, which thus serves as an internal model of the external posture of the limb.

spatial medium of the nervous system. More generally, it is possible to create a harmonic resonance mechanism whose standing wave pattern duplicates in mirror-image fashion the spatial pattern of deflection of the limb, in an analogical, or quasi-pictorial spatial model of the limb duplicated in the volume of the central nervous system. The exact coding scheme actually employed in vertibrates like the centipede is merely conjecture at this point. The purpose of this thought experiement is merely to suggest how a harmonic resonance could in principle serve to construct analogical spatiotemporal models, or effigies, of the limbs that they control, and how the modularity of harmonic resonance offers a mechanism for coupled spatial patterns. However the exact coding scheme that is actually used by vertibrates could be easily determined through electrophysiological recordings, once it is clear that it is the phase of oscillation that provides the significant signal to be observed. This hypothesis is testable neurophysiologically.

The advantage of an explicitly spatial principle of analogical representation is that it allows explicitly spatial computational interactions between the standing waves in the representation. For example, spatiotemporal interactions between the standing waves representing the left and right legs controlled by each ganglion could be synchronized by spatial interactions between their corresponding standing waves in the ganglion, to produce coordinated, symmetrical patterns of motion in the two opposing legs, like a swimmer’s breast stroke, as suggested in Figure 4.10, where the standing wave patterns in the spinal cord interact at the center where they meet. In fact, exactly this same principle of symmetrical motions coupled by standing wave resonances is seen already in some of the very simplest organisms, the flagella of the swimming Chlamydomonas.

Motor Control in Cerebrate Animals

A harmonic resonance theory of motor control seems clear enough in simple creatures like a centipede, with its identical series of legs moving in phase-lagged copies of the same motion. But what of the cerebrate creature, beginning with worms and centipedes, whose frontal ganglia are considerably larger than all the others, the first step towards a brain. What kind of processing takes place in this oversized frontal ganglion? The modularity principle in harmonic resonance theory suggests that the relation of the “brain” to the rest of the nervous system, is similar to the relation of the spinal nervous system to its peripheral nerves, which is a cyclic reciprocal reflection against each other in harmonic resonance. Pulses of electrical energy triggered at the boundary where the “brain” and spinal cord meet, propagate simultaneously down the spinal cord, and at the same time up into the brain, where the echo returns received back from the spinal cord are painted out in a spatiotemporal sweep, creating a mirror-image replica of the pattern of the spinal cord as a model in the brain, sculpted out of electrical standing waves. In the case of the centipede, the standing wave pattern in the “brain”, or frontal ganglion, would be a mirror-image reflection of the chain of ganglia of the whole creature, as suggested in Figure 4.11 A. Of course the speed of wave propagation in the brain would have to be considerably slower than that in the spinal cord, in order for the standing wave pattern to fit into the smaller dimensions of the brain.

The periodic pattern of echos returning from the chain of ganglia in response to a single centrifugal pulse, are painted out in sequence as a spatial pattern by the centripetal wave propagating up into the brain. This pattern of echos in turn can be used to lay out a pattern of nodes in the brain representation, between which a self-amplified standing wave can spontaneously emerge, creating a functional replica not only of the spatial pattern of the spinal cord and its periodic ganglia, but also of its resonant characteristics.

The brain-copy of the resonance of the ganglia would also include a copy of their resonances with their peripheral nerve, creating miniature traces of the limb nodes along each leg in the brain representation, reconstructing a spatial skeleton of the

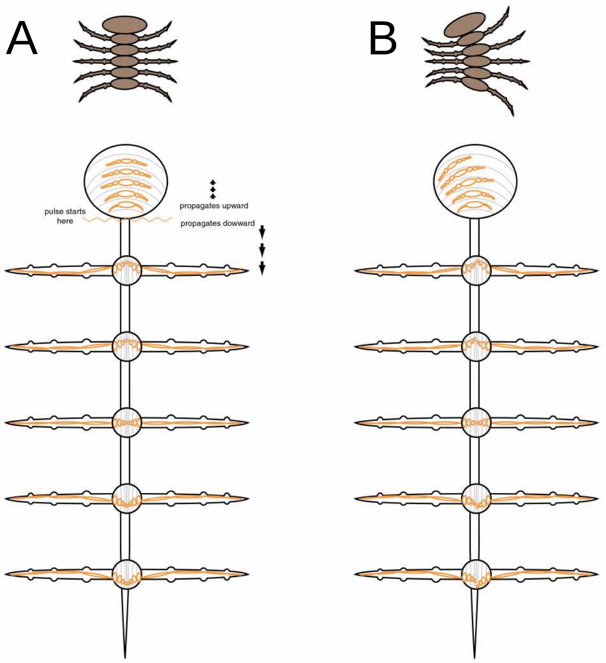

Figure 4.11 A: An electrical pulse, starting in the centipede's "brain" stem, propagates simultaneously down the spinal cord, and upward into the "brain". The periodic pattern of echos returning from the ganglia paint out a periodic spatial pattern in the creature's "brain", which in turn establishes a pattern of standing waves that mimick the dynamics of the spinal cord and peripheral nerves. B: Modulation of the global pattern in the "brain" warps that standing wave representation, for example to turn left, and that warp in the body image in the "brain" in turn warps the standing waves in the nervous system, causing the creature to actually turn left.

body in a spatial replica in the brain. In fact, the resonance goes one step farther, because the pulses emitted by the peripheral nerve at the center of each limb, propagates out radially through the limb, reflecting back from the muscle tissue enervated by that nerve, creating an echo of the outward pulse reflecting back from the outer surface of the enervated muscle, and this little echo would also be replicated in the spatial model of the body in the brain (not shown in Figure 4.11), where it would be plotted as an outer shell surrounding the replica of the peripheral nerve, with cyclic waves propagating radially from the periphery to the central nerve and back out again. In other words, what is constructed in the brain is a standing wave model of the body, complete with a spinal cord connecting periodic ganglia, with peripheral nerves connecting periodic nodes, with a peripheral body surface, all pulsing in-and-out in cyclic repetition, a dynamic spatiotemporal replica in the brain of the complex hierarchy of resonances pulsing through the nervous system and musculature. And that dynamic replica of the body in the brain remains functionally coupled to the body that it represents, such that changes in the sensory signal at any point in the body has immediate effects at the corresponding location of the brain map of that part of the body, just as changes in the brain model of the body have immediate effects back on the body.

For example a global command might be imposed on the brain map for the creature to turn left as suggested in Figure 4.11 B. This would require a phase-lagging of the standing waves at some points, and advancing them at others, in order to curve the body to the required posture for a left turn. The “turn left” thought takes its meaning from the fact that it is capable of imposing a “turn left” posture on the brain’s map of the body, and since this body map is dynamically coupled to the nervous system with which it is resonating, the phase modulation will be communicated from the brain to the spinal cord, where it will phase-lag the left side of the body while advancing the phase of the right, subtly warping the cyclic motion of every leg in accordance with a globally synchronized pattern.

Cerebral Cortex and its Maps

The modularity, or multiplicative principle of harmonic resonance also accounts for the multiple maps observed in the cortex in higher mammals, often disposed in mirror-symmetry across a sulcus. The largest such symmetry is the symmetry of the two cerebral hemispheres, coupled through the corpus callosum, suggesting a lateral left/right resonance reflecting back and forth across the medial plane. The next is the symmetry across the central sulcus, with the somatosensory homunculus on one side, and a mirror-image motor homunculus on the other side. In fact, the sensory / motor split continues all the way down the spinal cord, across the dorsal / ventral dimension, suggesting the most primal resonance of vertibrates as a periodic oscillation of sensory-motor-sensory-motor, oscillating in the dorso-ventral direction along the entire spinal cord and brain. There are also coupled oscillations between the various cortical maps, such as the primary, and secondary, and tertiary visual and somatosensory corticices, many of which are spatially adjacent and aligned in mirror symmetry to each other, suggesting a cortical oscillation back and forth across that axis of symmetry.

While the present theory does not predict with any precision exactly how these maps are organized or interconnected, the theory of harmonic resonance in the brain does make a number of specific predictions. It predicts that the phase of cortical oscillation carries the essential information of spatial location, and that movements of a stimulus across the visual field, or tactile stimulus across the skin, will be found to correspond to some waveform shifting in phase along with that translation. The theory predicts that adjacent, mirror-symmetrical cortical maps, starting with the primary somatosensory and motor cortex areas, will exhibit phasic sweeps of electrical activity going back and forth symmetrically across the symmetry axis that joins them. More generally, the harmonic resonance theory predicts that the electrical activation of the cortex is synchronized to a universal phasic resonance, of which the oscillations in individual cortical areas or individual neuroanatomical structures are higher harmonics. And most generally still, the harmonic resonance theory predicts that the spatial patterns of our experience correspond directly to spatial patterns in our nervous system, explicitly painted out in a cyclic resonance in the form of spatiotemporal standing waves, which serves to couple all the individual parts of the computational mechanism of the brain into a single integrated system in which information is exchanged continuously and reciprocally, bottom-up, top-down, left-right, front-back, in parallel between the various different brain areas.